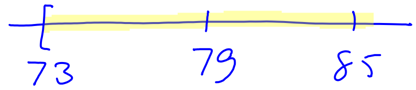
2013-11-13-C01

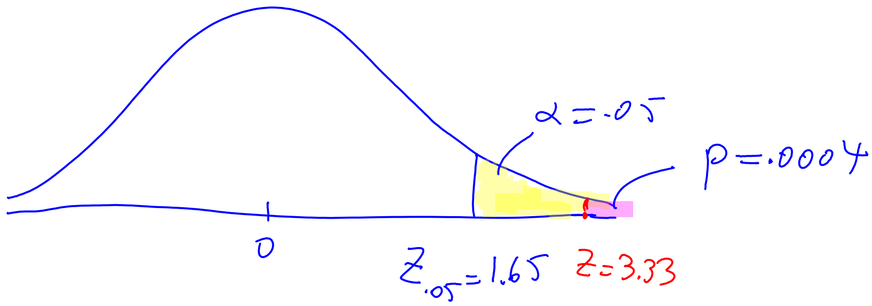
Wednesday, November 13, 2013
1:50 PM

95% CI for exam μ

$n=8$, $\bar{x}=79$, $s=7$



p vs. α



If $p<\alpha$, Then reject H_0

Ex. GMAT Scores

2005, $\mu=630$, $\sigma=45$

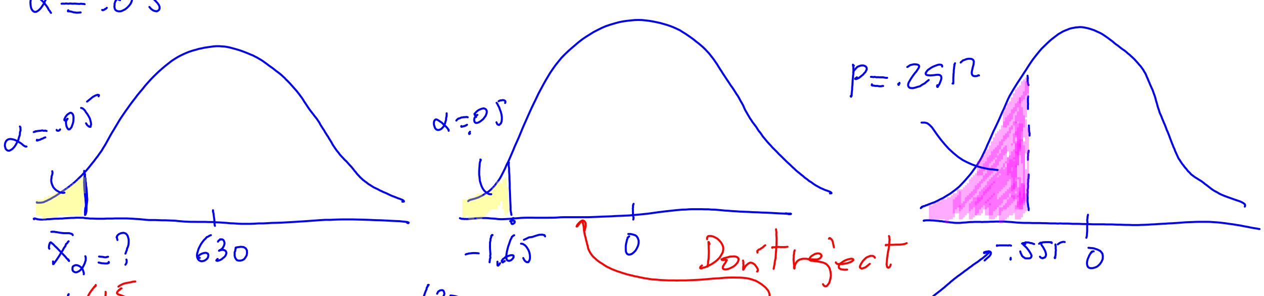
http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/GMAT-05.xls

For 2006, $H_0: \mu \geq 630$

$H_a: \mu < 630$

$n=25$, $\bar{x}=625$ $(\sigma=45)$ $\frac{\sigma}{\sqrt{n}}=\frac{45}{\sqrt{25}}=9$

$\alpha=.05$



$$z=\frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}=\frac{625-630}{9}=\boxed{-.555}$$

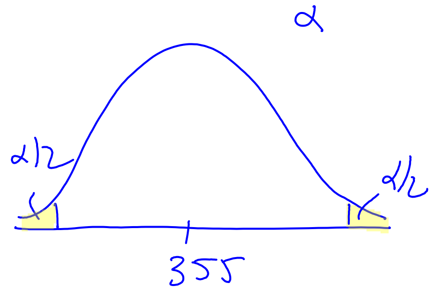
$\bar{x}_\alpha = ?$ $\qquad -1.65 = \dfrac{\bar{x}_\alpha - 630}{9} \Rightarrow \bar{x}_\alpha = 615$

Ex. Bottling $\qquad$ pb. 8-11, 8-45
p. 259, 274

Ideal 355 mL

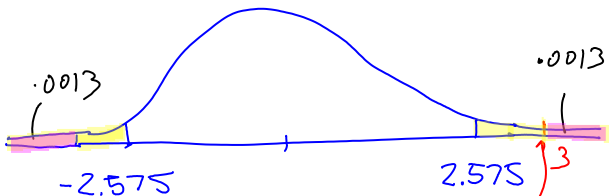
$H_0: \mu = 355$

$H_a: \mu \neq 355$



$\alpha = .01$, $n = 36$, $\sigma = .1$

$z_{\alpha/2} = 2.575$



$$z = \frac{\bar{x} - 355}{.1/\sqrt{36}}$$

a) $\bar{x} = 355.05$: $\quad z = \dfrac{355.05 - 355}{.1/\sqrt{36}} = \boxed{3}$ $\qquad$ Reject H_0

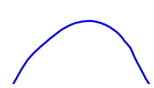
h2

$p = 2(.0013) = .0026 < \alpha = .01$ ✓

b) t-test about μ (σ unknown)
↓
use s & t (as in Ch. 7)

$H_0: \mu = \mu_0 \qquad t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$, $df = n - 1$

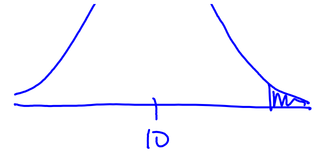
Ex. Tar content (cigarettes)



$H_0: \mu \le 10$

$H_a: \mu > 10$

σ unknown



$\alpha = .05$, $n = 10$

$\frac{s}{\sqrt{n}} = .4729$

9
11
10.5
12
9.5
10
10.5
9
8
13

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$\bar{x} =$

$s =$

10.0000	hypothesized value		
10.2500	mean Data		
1.4954	std. dev.		
0.4729	std. error		
10	n		
9	df		
0.53	t		
.3049	p-value (one-tailed, upper)		

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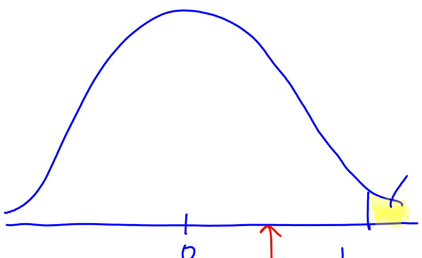
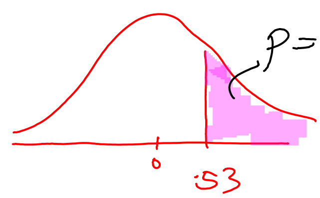
$t = \frac{10.25 - 10}{.4729} = .53$

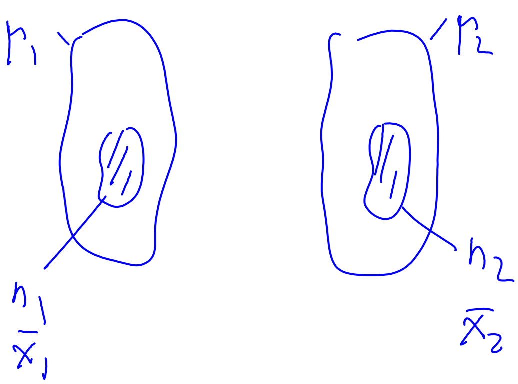


$t_{.05} = 1.833$

Don't reject $(p > \alpha)$



Ch. 9 Statistical inference (HT) based on two samples



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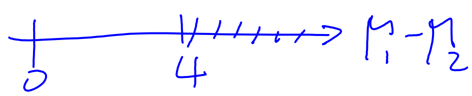
$H_0: \mu_1 - \mu_2 \ge D_0$



$H_a: \mu_1 - \mu_2 < D_0$

[9.1] z-test for $\mu_1 - \mu_2$

σ_1^2, σ_2^2 known

Recall

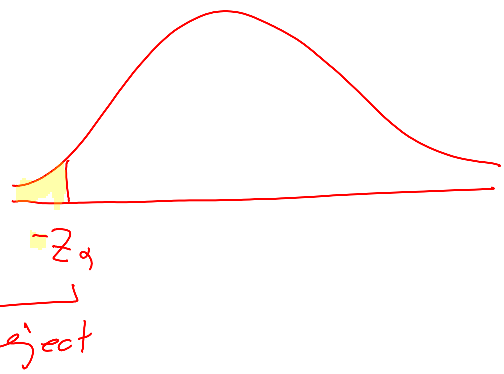
One pop

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

(Ch.8)

Two pop'n

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$ (Ch.7)



Ex. Atkins vs. conventional diet

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/Atkins-vs-Conventional-Diet-Class.xls

		Initial	6-month	Loss at			Initial	6-month	Loss at
	Diet	Weight (lbs)	Weight	6 Months (Atk)		Diet	Weight (lbs)	Weight	6 Months (Con)
1	Atkins	310	292.7	17.3	1	Conventional	256	259.5	-3.5
2	Atkins	309	275.1	33.9	2	Conventional	198	186.3	11.7
3	Atkins	257	217.7	39.3	3	Conventional	311	299.9	11.1
4	Atkins	227	221.1	5.9	4	Conventional	246	231.6	14.4
5	Atkins	231	204.5	26.5	5	Conventional	170	182.2	-12.2
6	Atkins	195	148	47	6	Conventional	244	251.2	-7.2
7	Atkins	190	179.7	10.3	7	Conventional	179	180	-1

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$H_0: \mu_1 - \mu_2 \geq 4$

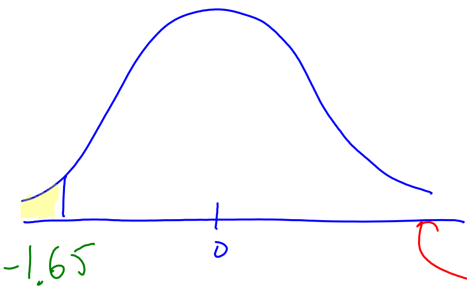
$H_a: \mu_1 - \mu_2 < 4$

A	C
$n_1 = 33$	$n_2 = 30$
$\bar{x}_1 = 15.42$	$\bar{x}_2 = 7$
$\sigma_1 = 8$	$\sigma_2 = 6$

$(\bar{x}_1 - \bar{x}_2) - D_0$ $8.42 - 4$

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{8.42 - 4}{\sqrt{\frac{8^2}{33} + \frac{6^2}{30}}} = \boxed{2.45}$$

$\alpha = .05$



Don't reject H_0!

Excluded material

	H_a
[9.2]	$\mu_1 - \mu_2 \neq D_0$
[9.3 & 9.4]	t-test $\mu_1 - \mu_2$
[9.5]	proportion

[9.6] F-test for equality of variances, i.e., $\sigma_1^2 = \sigma_2^2$

(In Ch. 7.6)

Ex. Diet problem, again

	6 Months (Atk)
count	33
mean	15.424
sample variance	206.528
sample standard deviation	14.371
minimum	-17.3
maximum	47
range	64.3

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$s_1^2 = (14.37)^2 = 206.52$

	6 Months (Con)
count	30
mean	7.007
sample variance	152.788
sample standard deviation	12.361
minimum	-12.9

$s_2^2 = (12.36)^2 = 152.78$

maximum	36.5
range	49.4

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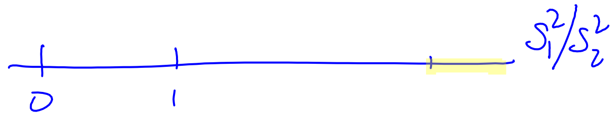
$s_1^2 > s_2^2$: does this imply $\sigma_1^2 > \sigma_2^2$

$H_0: \sigma_1^2 = \sigma_2^2$ | $H_0: \dfrac{\sigma_1^2}{\sigma_2^2} = 1$

$H_a: \sigma_1^2 > \sigma_2^2$ | $H_a: \dfrac{\sigma_1^2}{\sigma_2^2} > 1$

Intuition. Reject Ho if



3h

Result (Fisher). If H_0 is true, then

$\dfrac{s_1^2}{s_2^2}$ is F-distributed with

df_1: numerator $n_1 - 1$ | Atkins $33-1=32$

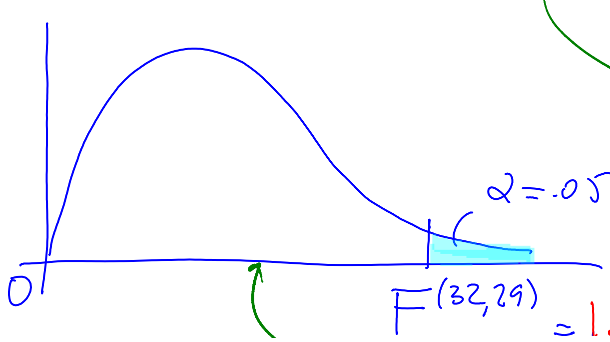
df_2: denominator $n_2 - 1$ | $30-1=29$

A	C
$n_1 = 33$	$n_2 = 30$
$s_1^2 = 206.5$	$s_2^2 = 152.78$
$df_1 = 32$	$df_2 = 29$

Test stat. $F = \dfrac{s_1^2}{s_2^2} = \dfrac{206.5}{152.78} = 1.35$

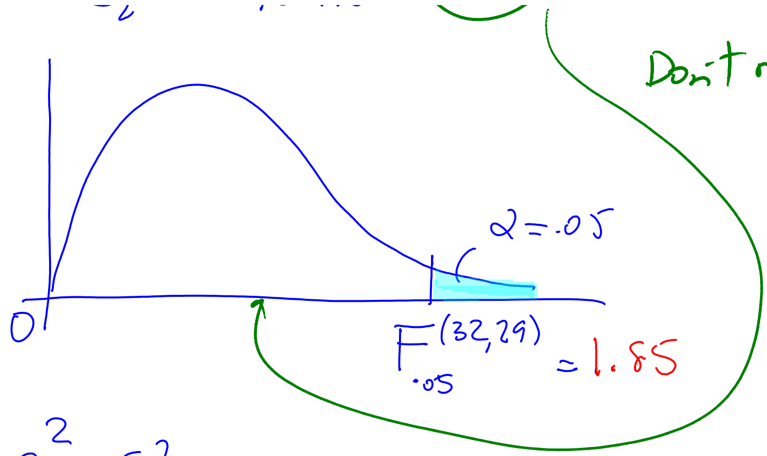
Don't reject

Let $\alpha = .05$



$= 1.85$

Let $\alpha = .05$



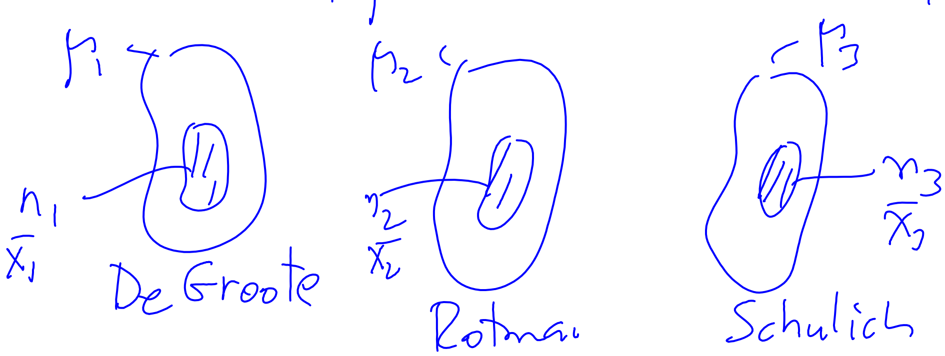
Remark If $s_2^2 > s_1^2$

$$\left.\begin{array}{l} H_0: \sigma_2^2 = \sigma_1^2 \\ H_a: \sigma_2^2 > \sigma_1^2 \end{array}\right) \quad F = \frac{s_2^2}{s_1^2}$$

Ch.10 Experimental design & analysis of variance (ANOVA)

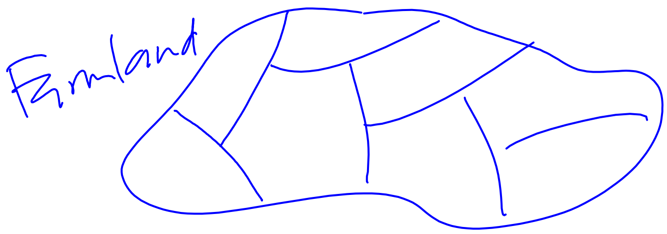
Three or more populations

$H_0: \mu_1 = \mu_2 = \mu_3$



a) Experimental design

Ex. Influence of fertilizer levels (L, M, H) on wheat yield



Factor (ind't var) $\xrightarrow{\text{influence}}$ dep't variable
- fertilizer   - yield
- moisture

Levels of factor(s): treatments $\longrightarrow$ yield level
L, M, H

Ex. Experimental farm

1 M	2 H	3 L	4 H	5 M	6 L
7 H	8 L	9 H	10 M	11 L	12 H
13 M	14 M	15 L	16 M	17 H	18 L

What we did: Completely randomized one-way experimental design (single factor with $k=3$ levels)

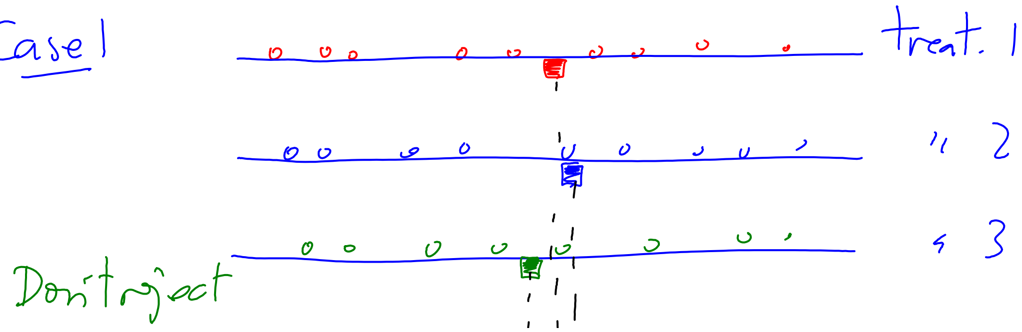
b) One-way ANOVA

Ex. Three pop'ns

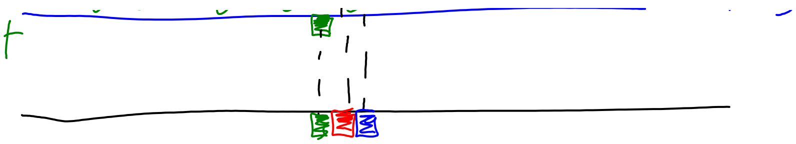
$H_0: \mu_1 = \mu_2 = \mu_3$

H_a: at least two differ

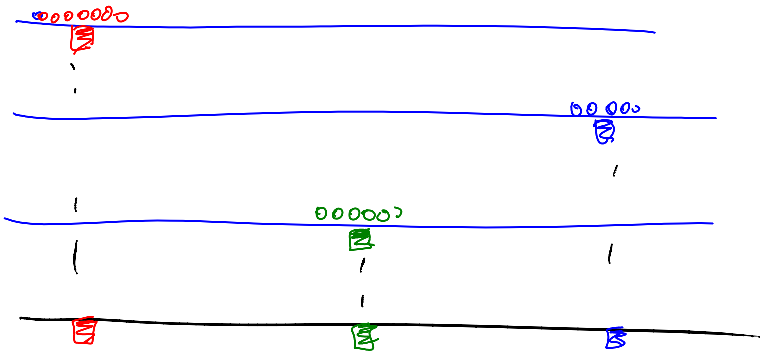
Case 1



Don't reject



Case 2



Reject

Case 3

Use ANOVA