2013-11-05-EC01

Tuesday, November 05, 2013
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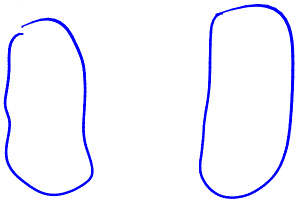
e) [7.5] CI for $\mu_1 - \mu_2$: Ind't samples σ_1^2, σ_2^2 known

μ_1 $\qquad\qquad$ μ_2



n_1 $\qquad\qquad$ n_2

σ_1^2 $\qquad\qquad$ σ_2^2

$\bar{x}_1$ $\qquad\qquad$ $\bar{x}_2$

[7.1] Single pop'n

$\bar{x}$ $\qquad$ α $\qquad$ $\mu_{\bar{x}} = \mu$ $\qquad$ $\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$

$100(1-\alpha)\%$ CI for μ

$$\left[\bar{x} \mp z_{\alpha/2} \frac{\sigma}{\sqrt{n}}\right]$$

$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ $\qquad$ $\bar{x}$ $\qquad$ $\bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

$\bar{X}_1 - \bar{X}_2$: diff in samples is normal with

$$\mu_{\bar{X}_1 - \bar{X}_2} = E(\bar{X}_1 - \bar{X}_2) = \mu_1 - \mu_2$$

$$\sigma_{\bar{X}_1 - \bar{X}_2}^2 = Var(\bar{X}_1 - \bar{X}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

$100(1-\alpha)\%$ CI for $\mu_1 - \mu_2$

$$\left[(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right]$$

✓

Ex Atkins vs. conventional diet

		Initial	6-month	Loss at				Initial	6-month	Loss at	
	Diet	Weight (lbs)	Weight	6 Months (Atk)			Diet	Weight (lbs)	Weight	6 Months (Con)	
1	Atkins	310	292.7	17.3		1	Conventional	256	259.5	-3.5	
2	Atkins	309	275.1	33.9		2	Conventional	198	186.3	11.7	
3	Atkins	257	217.7	39.3		3	Conventional	311	299.9	11.1	
4	Atkins	227	221.1	5.9		4	Conventional	246	231.6	14.4	
5	Atkins	231	204.5	26.5		5	Conventional	170	182.2	-12.2	
6	Atkins	195	148	47		6	Conventional	244	251.2	-7.2	
7	Atkins	190	179.7	10.3		7	Conventional	179	180	-1	

✓ U. of Penn

8	Atkins	164	131.1	32.9	8	Conventional	204	202.1	1.9	
9	Atkins	190	162.7	27.3	9	Conventional	261	265.5	-4.5	
10	Atkins	140	125.2	14.8	10	Conventional	271	260.3	10.7	
11	Atkins	251	240.8	10.2	11	Conventional	265	263	2	
12	Atkins	213	200.9	12.1	12	Conventional	262	266.8	-4.8	

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	6 Months (Atk)
count	33
mean	15.424
sample variance	206.528
sample standard deviation	14.371
minimum	-17.3
maximum	47
range	64.3

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	6 Months (Con)
count	30
mean	7.007
sample variance	152.788
sample standard deviation	12.361
minimum	-12.9
maximum	36.5
range	49.4

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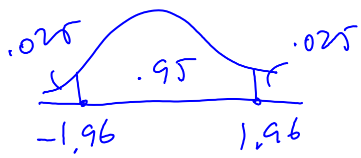
Weight loss (after 6 months)

Atkins	Conv
$n_1 = 33$	$n_2 = 30$
$\bar{x}_1 = 15.42$	$\bar{x}_2 = 7.00$
$\sigma_1 = 8$	$\sigma_2 = 6$
$\mu_1 = ?$	$\mu_2 = ?$

CI for $\mu_1 - \mu_2$: $\bar{x}_1 - \bar{x}_2 = 8.42$

95%

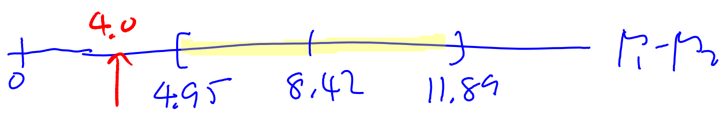


Atkins claims that on average their customers can lose 4 lbs. more than conventional

$$\left[(\bar{x}_1 - \bar{x}_2) \mp z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right]$$

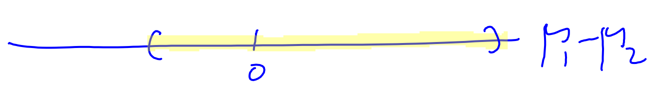
$$= \left[8.42 \mp 1.96\sqrt{1.94 + 1.20}\right]$$

$$= [4.95, 11.89]$$



Seems like Atkins more effective

Suppose

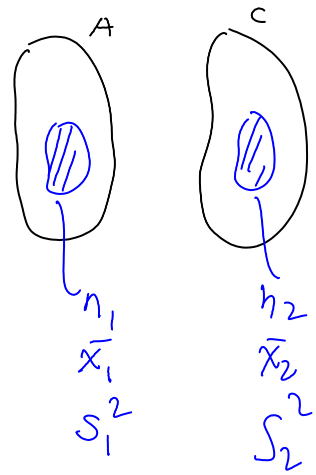


Not conclusive!

f) [7.6] CI for $\mu - \mu$: Ind't Sampls

f) [7.6] CI for $\mu_1 - \mu_2$: Ind't Samples

σ_1^2, σ_2^2 unknown but equal $\sigma_1^2 = \sigma_2^2 = \sigma^2$



Pooled estimate for σ^2

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$$

n2

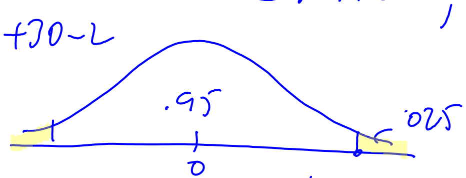
CI: $\left[(\bar{x}_1 - \bar{x}_2) \mp t_{\alpha/2}\sqrt{s_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}\right]$

$df = n_1 + n_2 - 2$

Ex. A vs. C

$n_1 = 33$	$n_2 = 30$
$\bar{x}_1 = 15.42$	$\bar{x}_2 = 7.00$
$s_1 = 14.37$	$s_2 = 12.36$

$$s_p^2 = \frac{(32)(206.50) + (29)(152.77)}{33+30-2} = 73.72, \quad df = 33+30-2 = 61$$

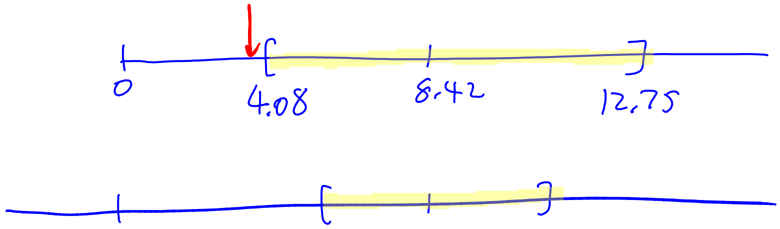


$t_{.025} = 2$

95% CI

$$\left[8.42 \mp 2.0\sqrt{73.72\left(\frac{1}{33}+\frac{1}{30}\right)}\right] = [4.08, 12.75]$$

Atkins claim



σ_1^2, σ_2^2 unknown

σ_1^2, σ_2^2 known

0 4.95 8.42 11.89

Ch. 8 Hypothesis testing

		Actual C	P	
Pick	C	3	1	4
	P	1	3	4
		4	4	

	C	P	
C	4	0	4
P	0	4	4
	4	4	

Pr(all correct) = 1.4%

a) Null & alternative hypotheses

Ex. legal Process: Someone charged with a crime. Every one is innocent until proven guilty

H_0: person innocent

H_a: " guilty

Actions. ① Reject H_0

② Accept (don't reject) H_0

		innocent H_0 true	guilty H_0 false
Decision	found guilty Reject H_0	Type I error	Correct
	" innocent Accept H_0	Correct	Type II error

$\alpha = Pr(\text{Type I error}) = Pr(\text{reject } H_0 \mid H_0 \text{ true}) =$ Steven Truscott

$\alpha = Pr(\text{Type I error}) = Pr(\text{reject } H_0 \mid H_0 \text{ true})$: Steven Truscott

h3 $\beta = Pr(\text{Type II} \text{ ''}) = Pr(\text{Accept } H_0 \mid H_0 \text{ false})$: O.J. Simpson

Intuition for rejecting H_0

Suppose claim is $H_0: \mu \le \mu_0$

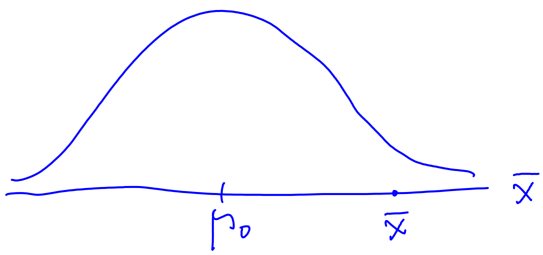
$H_a: \mu > \mu_0$

Take a sample $\rightarrow$ find $\bar{x}$

Ch. 6

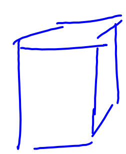
$\mu_{\bar{x}} = \mu_0$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$



If $\bar{x}$ is very high $\rightarrow$ reject H_0

Ex. Tar content of cigarettes



- Manuf Claim $H_0: \mu \le 10$
- Health group " $H_a: \mu > 10$

(One-sided H_a: >)

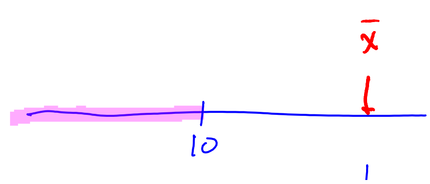
- Suppose $n = 25$ tested

$\sigma = 3$ mg , $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3}{\sqrt{25}} = .6$

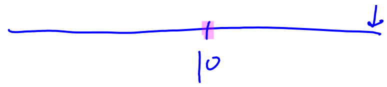
- $\alpha = .05$

- Select a test statistic

  - Replace $H_0: \mu \le 10$



with $H_0: \mu = 10$

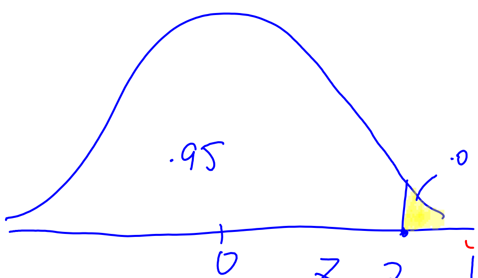


· find $\bar{x}$

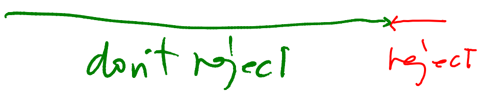
· $z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$, $\mu_0 = 10$: test statistic

· Or, use $\bar{x}$

· find a rejection point z_α & region for given α
or " " " " $\bar{x}_\alpha$ & " " " "

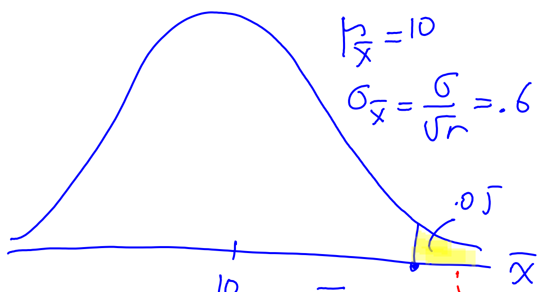


$z_\alpha = z_{.05} = 1.65$



$\mu_{\bar{x}} = 10$

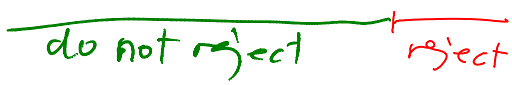
$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = .6$



$\bar{x}_\alpha = \bar{x}_{.05}$

$= \mu_0 + z_\alpha \frac{\sigma}{\sqrt{n}}$

$= 10 + 1.65(.6) = 10.99$



· Collect data & find

$\bar{x} = 12$

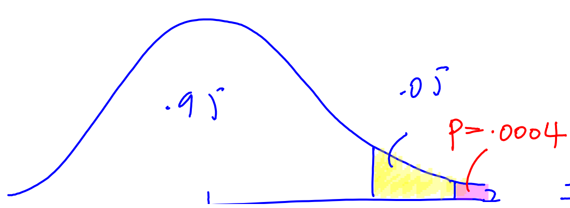
reject

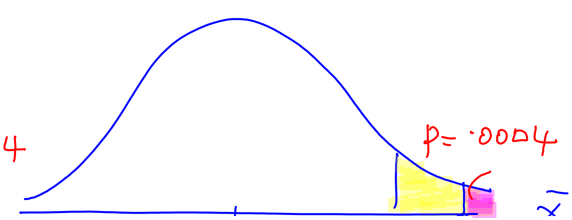
reject

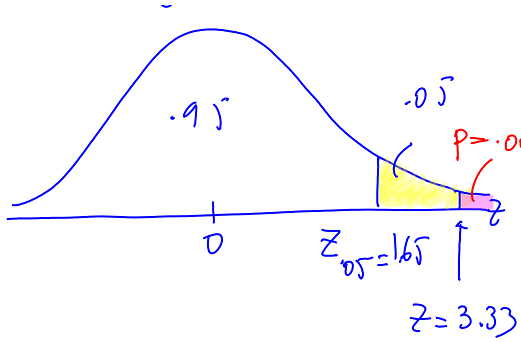
$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{12 - 10}{3/\sqrt{25}} = 3.33$$

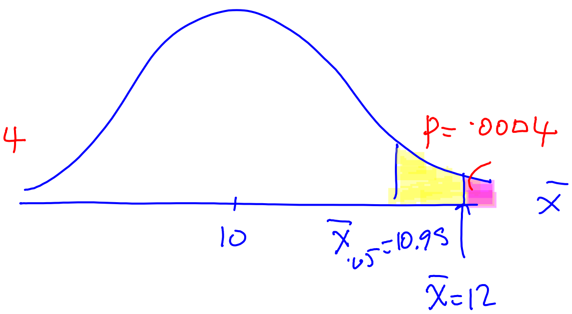
The p-value for testing hypothesis

Ex. Cig's

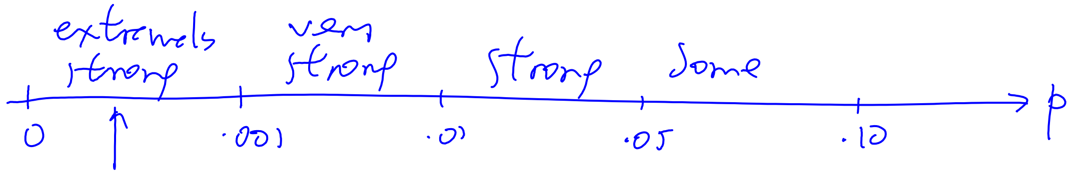








Evidence to reject H_0



.0004 → reject H_0

Small p values
are "bad news" for H_0