Ch 6 $\bar{X}$ $\mu_{\bar{X}} = \mu$, $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$

Ch. 7. Confidence Intervals (CI)

$\mp 3\%$ 31   34   37 $\hat{p}$ 95%

$\frac{19}{20} = \uparrow$

$\mp 2\%$ 32   34   36

a) [7.1] z-based CI for μ (σ known)

Ex. Physicians' taxable incomes

($100K)

	x	$p(x)$
possible taxable incomes	0	?
	1	?
	2	?
	3	?
	4	?

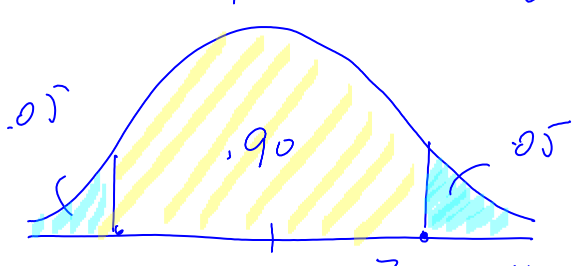
$\mu = ?$

$\sigma = 1.26$ (known)

$n = 5$: 3, 0, 3, 0, 3 | $\bar{x} = 1.8$ ($180,000)

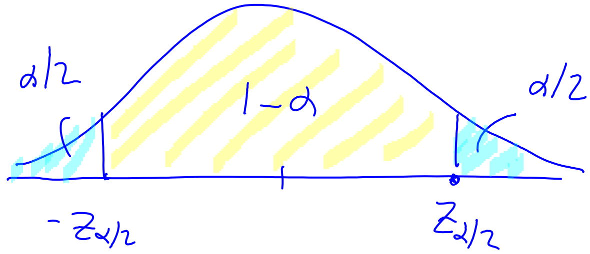
Recall ① Pop'n is normal
or ② sample large } $\Rightarrow \bar{X}$ normal

90%



$\alpha = .10$

$-z_{.05} = -1.64 \qquad 0 \qquad z_{.05} = 1.64$

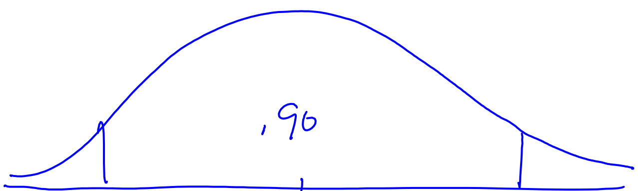


Recall $\quad X$ normal $\quad \mu_X = \mu, \quad \sigma_X^2 = \sigma^2$

n sample $\quad \bar{X}$ " $\quad \mu_{\bar{X}} = \mu, \quad \sigma_{\bar{X}}^2 = \dfrac{\sigma^2}{n}$ $\Bigg\}$ Ch. 6

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

90% CI



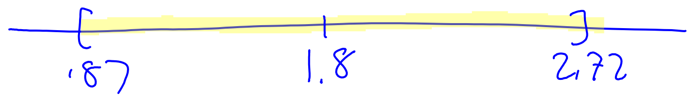
$$-1.64 = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \qquad 0 \qquad 1.64 = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\mu = \bar{x} - 1.64 \frac{\sigma}{\sqrt{n}} \qquad\qquad \mu = \bar{x} + 1.64 \frac{\sigma}{\sqrt{n}}$$

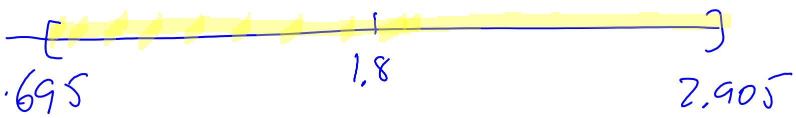
$$\mu = 1.8 - 1.64 \frac{1.26}{\sqrt{5}} \qquad\qquad \mu = 1.8 + 1.64 \frac{1.26}{\sqrt{5}}$$

$$= .87 \qquad\qquad = 2.72$$

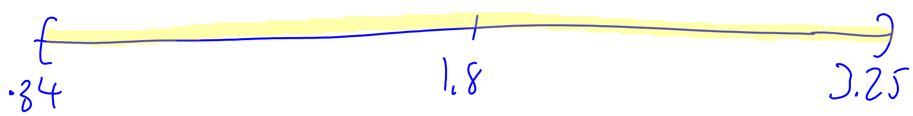


90%

95%



99%



h2 For $(1-\alpha)$ CI [$100(1-\alpha)\%$ CI]

$$-z_{\alpha/2} = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \qquad z_{\alpha/2} = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$$

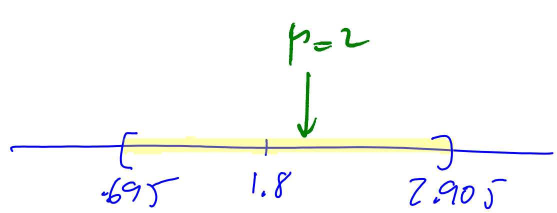
$$\Downarrow \qquad\qquad \Downarrow$$

$$\mu = \bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \qquad \mu = \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

Meaning of 95% ? etc

3, 0, 3, 0, 3

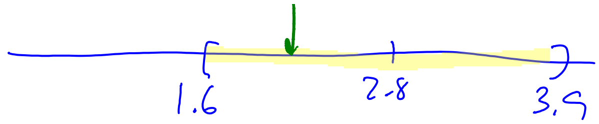
$\bar{x} = 1.8$



$\mu=2$

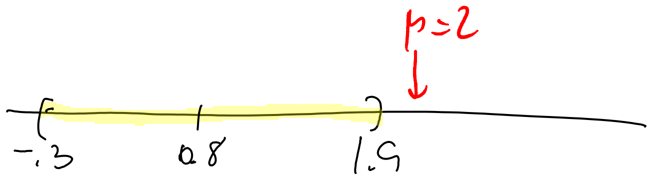
4, 4, 3, 2, 1

$\bar{x} = 2.8$



2, 0, 2, 0, 0

$\bar{x} = 0.8$



Revealed.

Popn

x	$p(x)$
0	.1
1	.2
2	.4
3	.2
4	.1

$\mu = \sum x\, p(x) = 2$

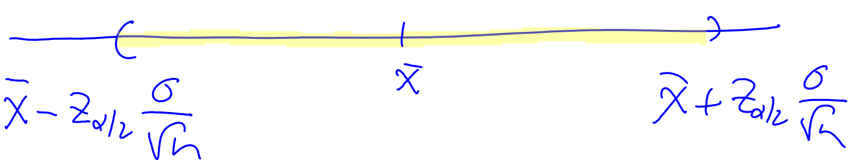
Ex. Visual stats

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/CI-2.wmf

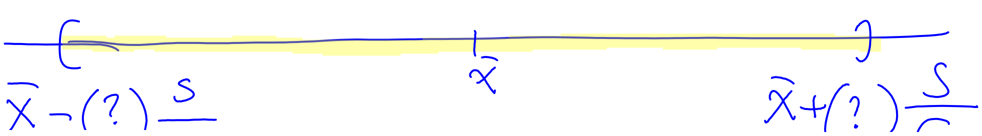
(h) [7.2] t-based CI for μ (σ unknown)

$$\sigma^2 \longrightarrow s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$$

σ known



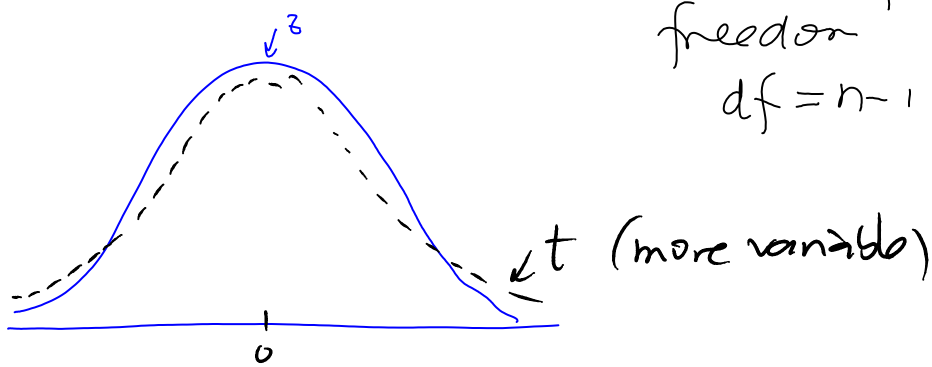
σ unknown

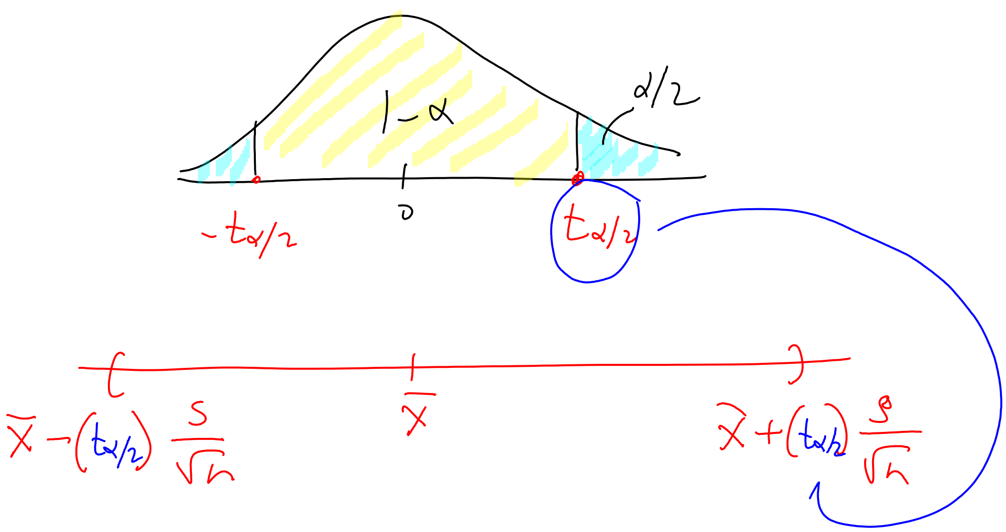


$\sqrt{n}$ $\sqrt{n}$

If σ known $\quad Z = \dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}}$

If σ unknown $\quad T = \dfrac{\bar{X} - \mu}{S/\sqrt{n}}$: t-random variable with degrees of freedom $df = n-1$





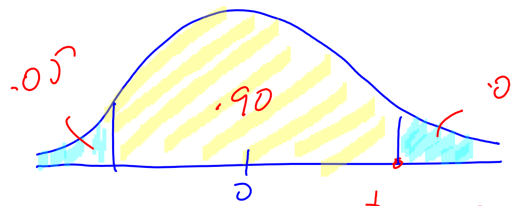
Ex. Car mileage

$n = 6$. 18.6, 18.4, 19.2, 20.8, 19.4, 20.5

90% CI

$\bar{x} = 19.48$

$s^2 = 1.12$, $S = 1.06$, $\dfrac{s}{\sqrt{n}} = 0.43$



$t_{.05} = 2.015$ $df = 6-1 = 5$

h3

18.61 19.48 20.35

$\bar{x} - t_{\alpha/2}\frac{s}{\sqrt{n}} = 19.48 - 2.015(.43)$

$\bar{x} + t_{\alpha/2}\frac{s}{\sqrt{n}} = 19.48 + 2.015(.43)$

Exercise. If we had known that $\sigma = 1.06$

18.77 19.48 20.19

c) [7.3] What is the best sample size n? (σ known)

error
$E = z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ $\bar{x}$ $\bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

$$E = z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

Fix E, solve for n

$$n = \left(\frac{z_{\alpha/2}\cdot\sigma}{E}\right)^2$$

Ex. Taxable income (again!)

$n=5$, $\bar{x} = 1.8$, $\sigma = 1.26$, $z_{\alpha/2} = 1.96$

$n=5$, $\bar{x}=1.8$, $\sigma=1.26$, $z_{\alpha/2}=1.96$

95%

$.695 \quad 1.8 \quad 2.905$

$\underbrace{1.8 \quad 2.905}_{E=1.105}$ ($110,500)

$E=.5$ ($50,000)

$$n=\left(\frac{z_{\alpha/2}\cdot\sigma}{E}\right)^2=\left(\frac{1.96\times1.26}{.5}\right)^2=24.39\to25$$

$E=.5$

$E=.1$

$10,000

$$n=\left(\frac{1.96\times1.26}{.1}\right)^2=609$$

Remark What if σ unknown?

- Take a preliminary sample size m
- Estimate s
- Use $t_{\alpha/2}$ with $df=m-1$

$$n=\left(\frac{t_{\alpha/2}\cdot s}{E}\right)^2$$

Ex. Car mileage 90%

$m=6$, $s=1.06$, $t_{.05}=2.015$ $\Big\}$ $E=.87$ $\quad .87$

$df=5$

$E=.4$: $\quad n=\left(\frac{2.015\times1.06}{.4}\right)^2=28.51\to29$ $\quad .40$

(d) [7.4] CI for a proportion p

(d) [7.4] CI for a proportion p

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \sqrt{\frac{1}{n} p(1-p)} \longrightarrow \sigma_{\hat{p}} = \sqrt{\frac{1}{n} \hat{p}(1-\hat{p})}$$

$$\left[\ \hat{p} - z_{\alpha/2}\sqrt{\frac{1}{n}\hat{p}(1-\hat{p})} \qquad \hat{p} \qquad \hat{p} + z_{\alpha/2}\sqrt{\frac{1}{n}\hat{p}(1-\hat{p})}\ \right]$$

Ex. Globe (prop. of water to total surface area)

True = 71%

W W W L W
L W W L W
W W W L W
W L W W W
W L L W W
L L W W L

$$\hat{p} = \frac{20}{30} = .68$$

Confidence interval - proportion		
	95%	confidence level
	0.67	proportion
	30	n
	1.960	z
	0.168	half-width
	0.838	upper confidence limit
	0.502	lower confidence limit

p = .71

[.502 — .67 — .838]

Q: What's the best sample size n?

$$n = 0.25\left(\frac{z_{\alpha/2}}{E}\right)^2$$

Ex. Election

$E = 0.035$

95% CI $\Rightarrow$ $z_{\alpha/2} = 1.96$

$$n = .25\left(\frac{1.96}{.035}\right)^2 = 784 \simeq 816$$

If $E = .01$

$$n = 9,604 \quad \text{(too many)}$$