2013-10-23-C01

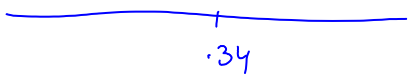
Wednesday, October 23, 2013

11:26 AM

No class next week

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/LectureNotes/documents/2013-10-16-C01.pdf

Ch. 7 Confidence Intervals (CI)



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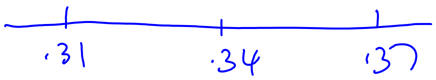


(a) [7.1] z-based CI for μ (σ known)

Ex. Physician Salaries

Auditor

x	$p(x)$
0	?
1	?
2	?
3	?
4	?

($100K)

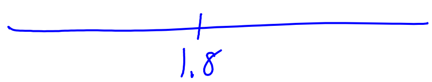
$\mu = ?$

$\sigma = 1.26$

x_1	3
x_2	0
x_3	3
x_4	0
x_5	3

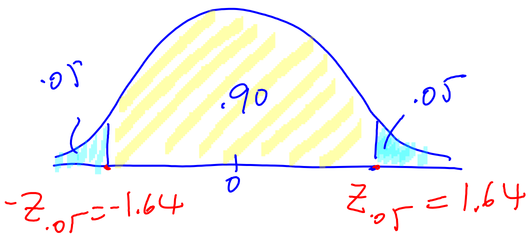
$\bar{X} = 1.8$

($180,000)



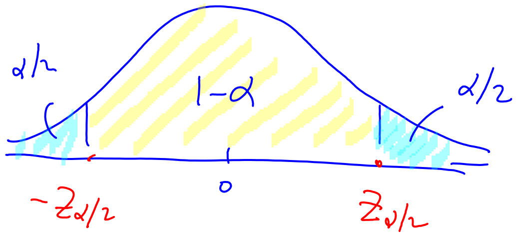
Recall ① pop'n is normal or ② sample size large $\Rightarrow$ $\bar{X}$ normal

Recall Ch. 5 90% prob for Z



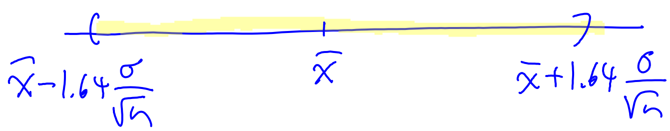
$Pr(-1.64 \le Z \le 1.64) = .90$

$\alpha = .10$



$Pr(-z_{\alpha/2} \le Z \le z_{\alpha/2})$
$= 1-\alpha$

Claim A 90% CI for μ is as follows



$1.8-1.64\frac{1.26}{\sqrt{5}} = .87$ $\qquad$ $1.8+1.64\frac{1.26}{\sqrt{5}} = 2.72$

How? $\quad Z = \frac{X-\mu}{\sigma}$

Similarly $\quad Z = \frac{\bar{X}-\mu}{\sigma_{\bar{X}}} = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$

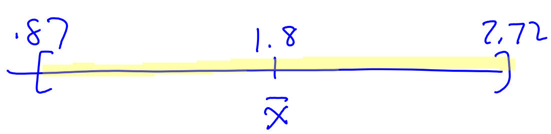
$$Pr\left(-1.64 \le Z \le 1.64\right) = .90$$

$$\Downarrow$$

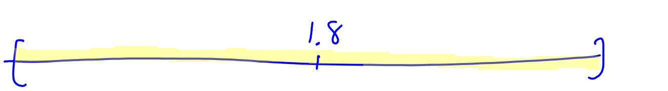
$$-1.64 \le \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \le 1.64$$

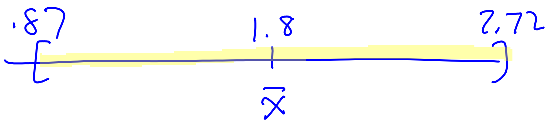
$$-1.64\frac{\sigma}{\sqrt{n}} \le \bar{X}-\mu \le 1.64\frac{\sigma}{\sqrt{n}}$$

$$Pr\left(\bar{X}-1.64\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{X}+1.64\frac{\sigma}{\sqrt{n}}\right) = .90$$

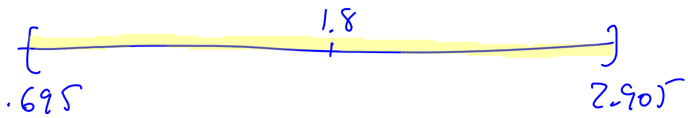


95% CI

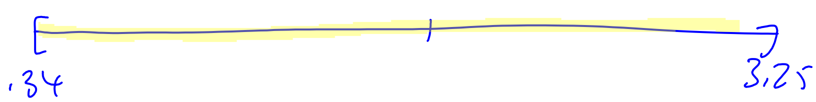




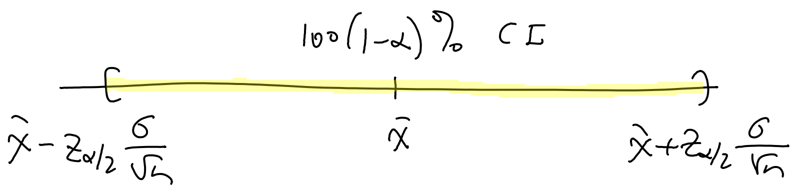
95% CI



99%



General

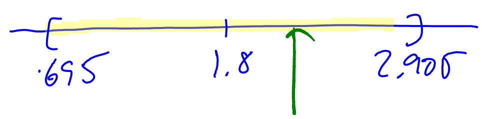


What do these mean?

Consider 95% case

$\bar{x} = 1.8,\ \sigma = 1.26,\ \alpha = .05,\ z_{.025} = 1.96$

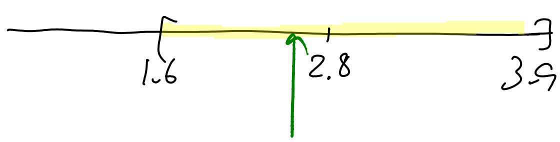
3, 0, 3, 0, 3



New sample

4, 4, 3, 2, 1

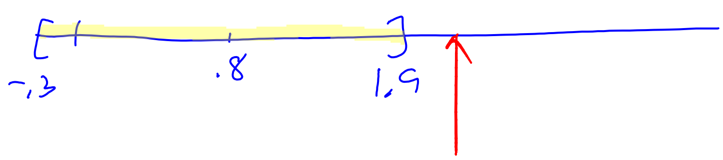
$\bar{x} = 2.8$



New sample

2, 0, 2, 0, 0

$\bar{x} = 0.8$



Reveal

x	$p(x)$
0	.1
1	.2
2	.4
3	.2
4	.1

$\mu = 2$

Visual statistics

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/CI-2.wmf

b) [7.2] t-based CI for μ (σ unknown)

$\bar{x} \to \mu$

$s^2 \to \sigma^2$

$$s^2 = \frac{1}{n-1}\sum (x_i - \bar{x})^2$$

$$s = \sqrt{s^2}$$

If σ known

$$\left(\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \quad , \quad \bar{x} \quad , \quad \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \right)$$

If σ unknown

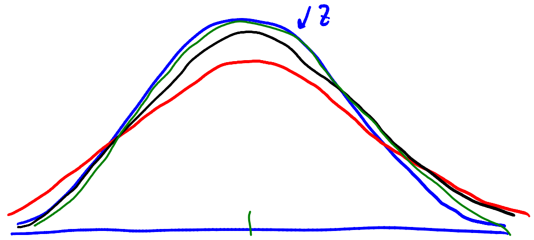
$$\left(\bar{x} - (?)\frac{s}{\sqrt{n}} \quad , \quad \bar{x} \quad , \quad \bar{x} + (?)\frac{s}{\sqrt{n}} \right)$$

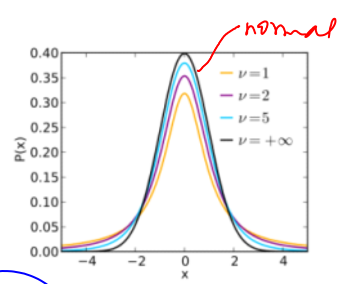
$? = t_{\alpha/2}$, $? = t_{\alpha/2}$

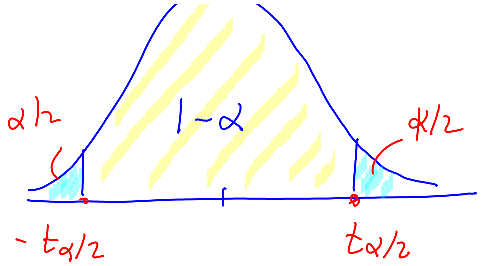
$$\cancel{Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}}$$

$$T = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

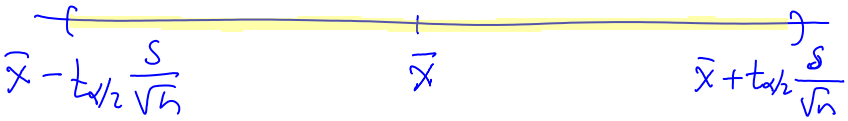
: t-distribution has degrees of freedom $df = n-1$







$df = n-1$



Ex. Random sample $n=6$ cars ← mileage

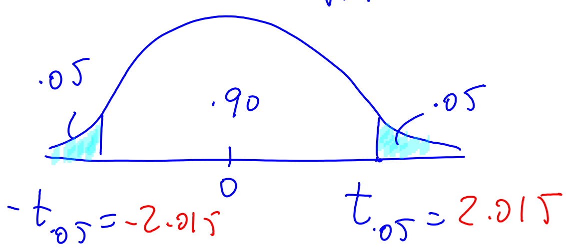
18.6, 18.4, 19.2, 20.8, 19.4, 20.5

$\bar{x} = \frac{1}{n}\sum x_i = 19.48$

90% CI (assume pop'n normal)

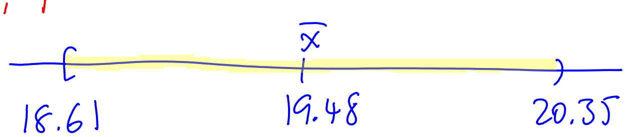
$$s^2 = \frac{1}{n-1}\sum (x_i - \bar{x})^2 = 1.12$$

$$s = \sqrt{s^2} = 1.06, \quad \frac{s}{\sqrt{n}} = 0.43$$



$df = n-1 = 5$

df	$t_{.1}$	$t_{.05}$	$t_{.025}$
1			
2			
3			
4			
5		2.015	
⋮			



(σ unknown)

§3 Suppose we <u>knew definitely</u> that $\sigma = 1.06$ (Same as $s = 1.06$)

$\sigma = 1.06$

$$\underset{\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}}{\overset{18.77}{(}} \quad \underset{19.48}{|} \quad \underset{\bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}}{\overset{20.19}{)}}$$

=

c) [7.3] What is the least sample size n? (σ-known)

Recall

$$\text{Error} = \boxed{E = z_{\alpha/2}\frac{\sigma}{\sqrt{n}}}$$

$$\underset{\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}}{|} \quad \underset{\bar{x}}{|} \quad \underset{\bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}}{|}$$

$$E = z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \implies n = \left(\frac{z_{\alpha/2}\cdot\sigma}{E}\right)^2$$

Ex. Taxable income

$n=5$, $\bar{x} = 1.8$, $\sigma = 1.26$. 95% $\quad z_{.025} = 1.96$

$$\underset{.695}{(} \quad \underset{1.8}{|} \quad \underset{2.905}{)}$$

$E = 1.105$ (\$110,500)

$E = .5$ (\$50,000)

$$n = \left(\frac{1.96 \times 1.26}{.5}\right)^2 = 24.39 \to 25$$

$E = .1$ (\$10,000)

$$n = \left(\frac{1.96 \times 1.26}{.1}\right)^2 = 609 \quad \text{too many}$$

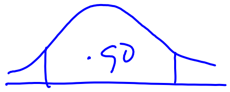
Remark What to do if σ unknown?

- Take a preliminary sample of size m
- Estimate s

- Use $t_{\alpha/2}$ with $df = m-1$

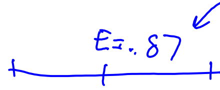
$$n = \left(\frac{t_{\alpha/2}\cdot s}{E}\right)^2$$

Ex. Mileage



$m=6$, $s^2 = 1.12$, $s = 1.06$, $t_{.05} = 2.015$ ← old

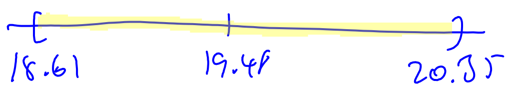
$df = 6-1 = 5$



$E = .4$, $n = \left(\frac{t_{.05} \times s}{E}\right)^2$

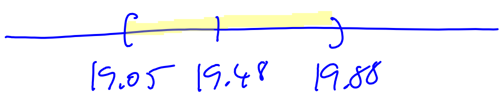
$$= \left(\frac{2.015 \times 1.06}{.4}\right)^2 = 28.51 \rightarrow 29$$

$E = .87$



$m = 6$

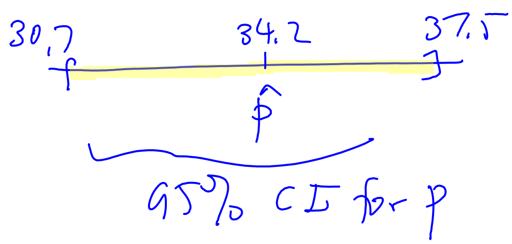
$E = .4$



$n = 29$

d) [7.4] CI for a proportion p

Election poll



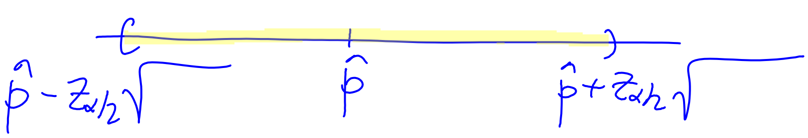
$\mp 3.5\%$

If n large

Ch. 6

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \sqrt{\tfrac{1}{n}p(1-p)} \longrightarrow \sqrt{\tfrac{1}{n}\hat{p}(1-\hat{p})}$$



$\hat{p} - z_{\alpha/2}\sqrt{\quad}$ ($\hat{p}$) $\hat{p} + z_{\alpha/2}\sqrt{\quad}$

Ex. Globe (Prop of water)

W			L	
5	~~||||~~		~~||||~~	5
5	~~||||~~		~~||||~~	5
5	~~||||~~		|	
3	|||			
18			11	

$$\hat{p} = \frac{18}{29} = .62$$

Confidence interval - proportion		
	95%	confidence level
	0.62	proportion
	29	n
	1.960	z
	0.177	half-width
	0.797	upper confidence limit
	0.443	lower confidence limit

✓
71.8
(.443 .62 .797)

Q: what's best sample size n?

$$n = 0.25\left(\frac{z_{\alpha/2}}{E}\right)^2$$

Ex. Election

$\mp 3.5\%$ $\rightarrow$ $E = 0.035$

$19/20$ $\rightarrow$ 95% CI: $z_{\alpha/2} = 1.96$

$$n = 0.25\left(\frac{1.96}{.035}\right)^2 = 784 \sim 816$$

$E = 0.01$, $n = 9{,}604$ too many