2013-10-09-C01

Wednesday, October 09, 2013
1:57 PM

$p = Pr(\text{Success})$

Binomial

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, \ldots, n$$

Ex. New drug adoption

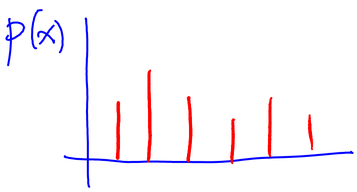
http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/Binomial-NewDrug.pdf

Ch. 5 Continuous random variables (r.v.)

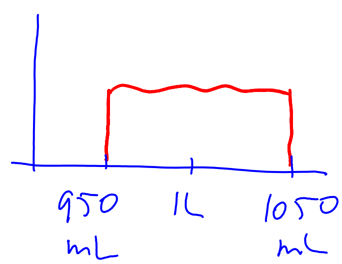
a) Continuous distribution

Random Variable

Discrete | Continuous



Sum of all heights = 1

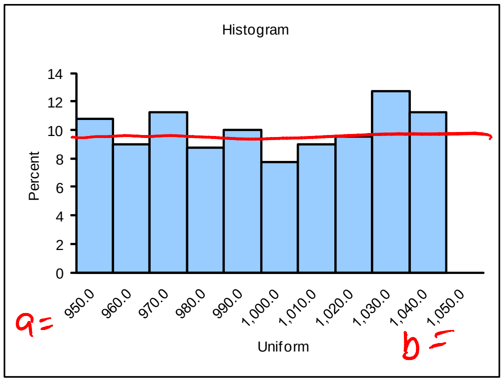


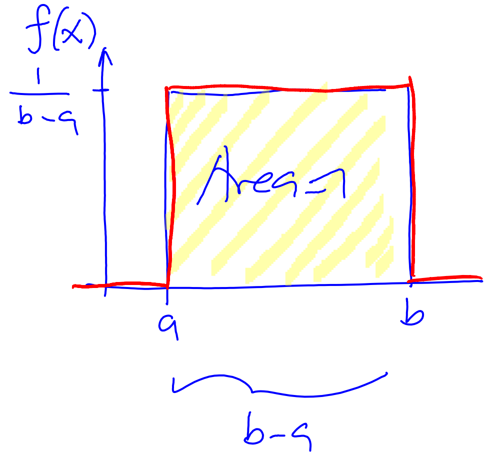
http://www.youtube.com/watch?v=dn1mIZS6I6o&feature=related

Area under curve = 1

b) Uniform distribution

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/AppleJuice.xls





$$f(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0, & \text{otherwise} \end{cases}$$

Mean $\mu_X = \frac{1}{2}(a+b)$

$$\sigma_X^2 = \frac{(b-a)^2}{12}$$

Ex. Apple juice $a = 950$, $b = 1050$, $\frac{1}{b-a} = .01$

±2% deviation from 1L is OK

$$Pr(980 \le X \le 1020) = 40 \times .01 = .4$$

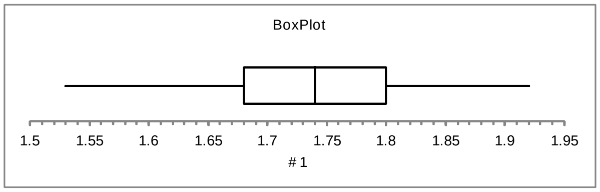
$$Pr(X = 990) = 0$$

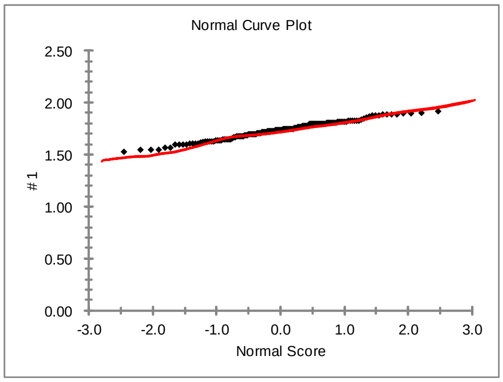
h2

c) Normal distribution

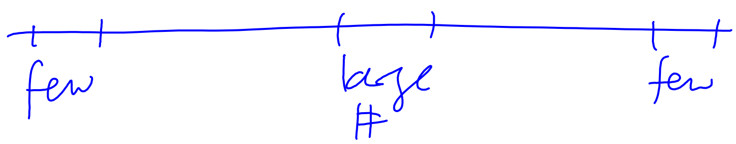
Ex. Heights data

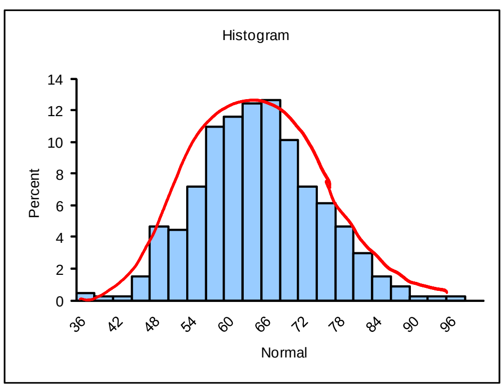
http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/Q600-2013-Scanned-Height-Gender-Handspan.pdf





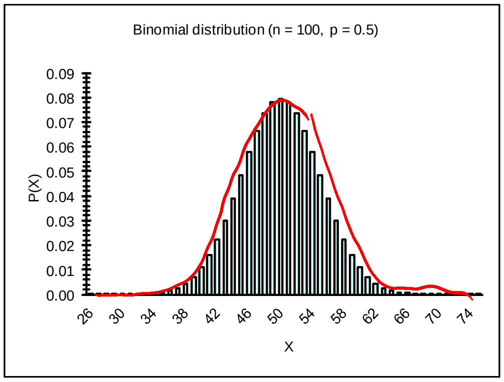
Ex. Test scores





Ex. Binomial with large n and $p \simeq 0.5$

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/Binomial-Normal.xls



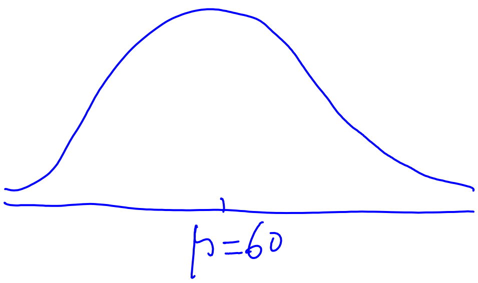
Ex. Galton board

http://mathworld.wolfram.com/GaltonBoard.html

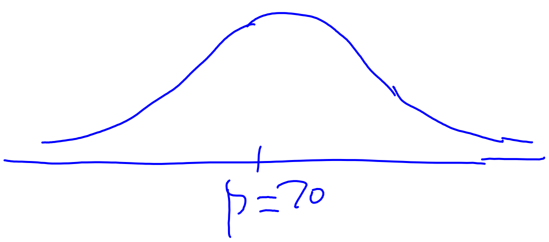
http://www.youtube.com/watch?v=xDIyAOBa_yU

What's the effect of μ and σ?

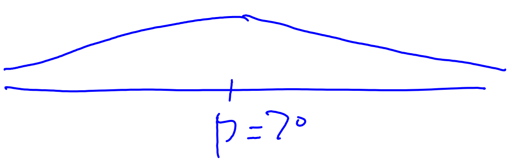
Ex.



$\mu=60, \quad \sigma=3$



$\mu=70, \quad \sigma=3$



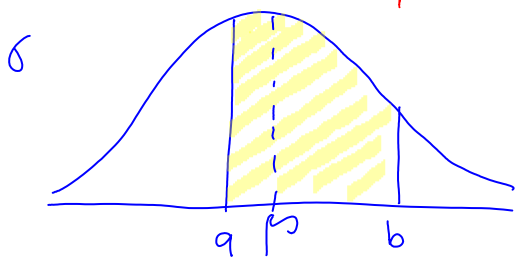
$\mu=70, \quad \sigma=6$

$\mu=70, \quad \sigma=0$

μ: characterizes location

σ: " dispersion

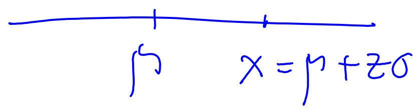
(i) Calculating probabilities



$\Pr(a \le X \le b) = ?$

Empirical rule is <u>useless</u>

<u>Z-Scores</u>



$$z = \frac{x - \mu}{\sigma}$$

If $z > 0$: x is above mean μ

If $z < 0$: x " below " μ

Define a new r.v. Z;

$$Z = \frac{X - \mu}{\sigma}$$

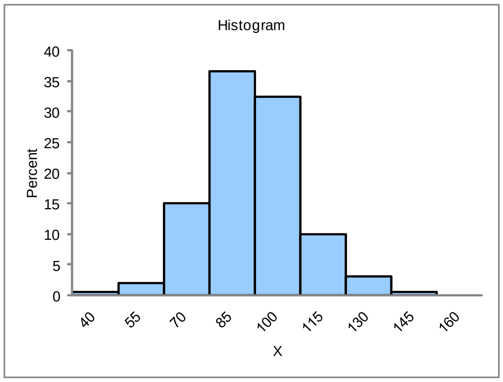
<u>Fact</u>

X is normal with mean μ and s.d. σ

Z " " " " 0 and " 1

<u>Demo</u>

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/IQ-Scores-200-2009.xlsx



	X
count	200
mean	99.0955
sample variance	233.0254
sample standard deviation	15.2652
minimum	47.65
maximum	148.69
range	101.04

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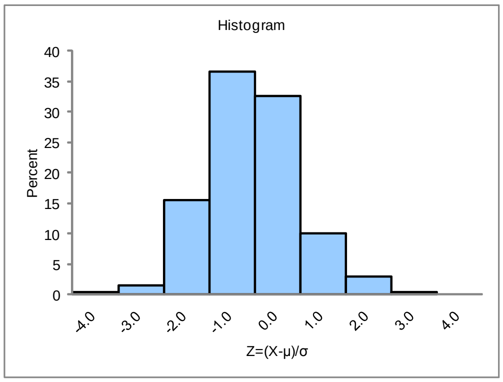

	Z=(X-μ)/σ
count	200
mean	-0.0603
sample variance	1.0357
sample standard deviation	1.0177
minimum	-3.49
maximum	3.246
range	6.736

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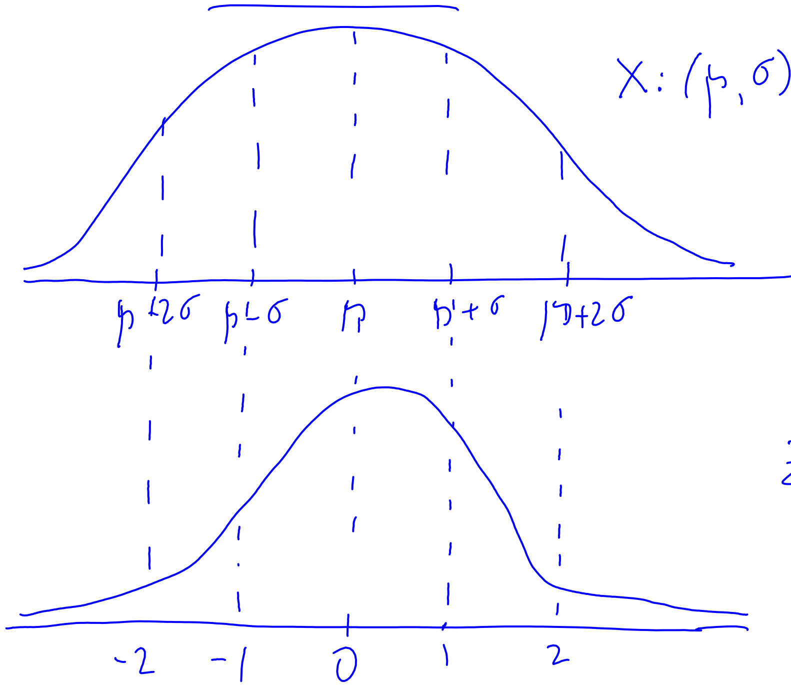
Z is called standardized normal r.v.



X: (μ, σ)

Z: (0,1)

Ex. Test X normal $\mu = 60$, $\sigma = 15$

$$\Pr(60 \le X \le 80) = ?$$

Standardization

$$60 \le X \le 80$$

$$\frac{60-60}{15} \le \underbrace{\frac{X-60}{15}}_{} \le \frac{80-60}{15}$$

$$\Pr(0 \le Z \le 1.33) = ?$$

3h

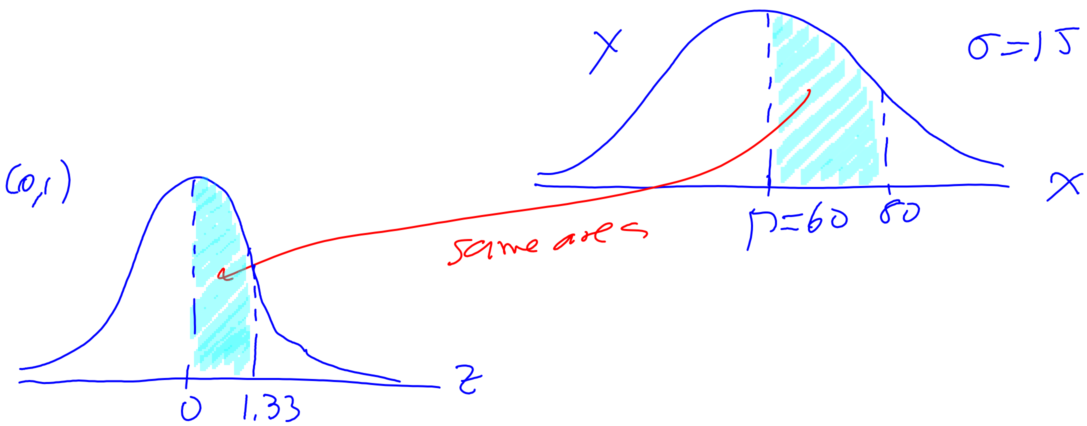
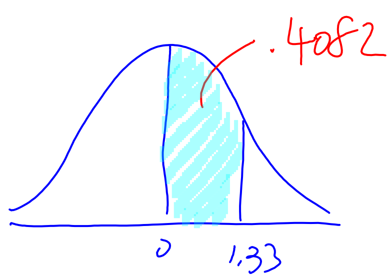


Table 5.1 (p. 156) or App. A.3 (p. 651)

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/NormalTable.wmf

Z	0.00	0.01	0.02	0.03
0.0				
0.1				
⋮				
1.0				
1.1				
1.2				
1.3				.4082
⋮				



$$\Pr(0 \le Z \le 1.33) = .4082$$

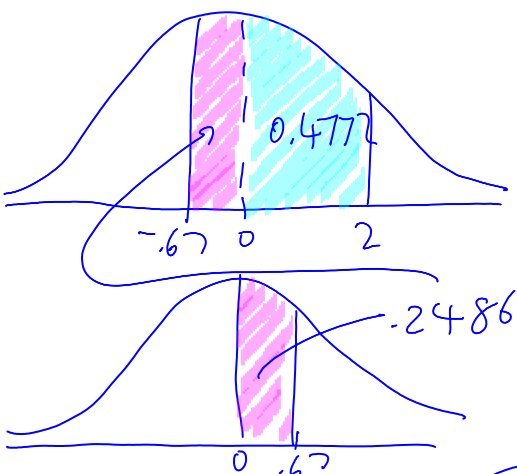
$$Pr(60 \le X \le 80) = .4082$$

Ex. $Pr(50 \le X \le 90) = ?$

$$50 \le X \le 90$$

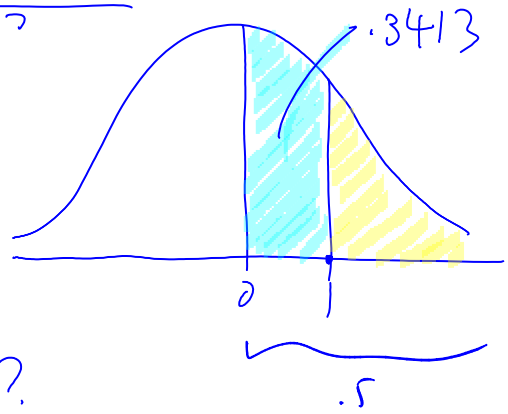
$$\frac{50-60}{15} \le \underbrace{\frac{X-60}{15}} \le \frac{90-60}{15}$$

$$Pr(-.67 \le Z \le 2) = \begin{array}{r} .4772 \\ .2486 \\ \hline .7258 \end{array}$$



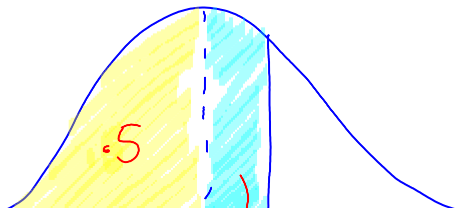
Ex. $Pr(Z > 1) = ?$

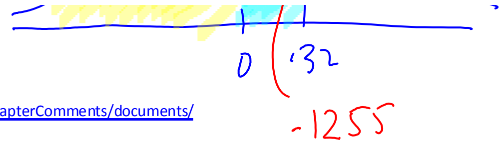
$= .5 - .3413 = .1587$



Ex. $Pr(Z \le 0.32) = ?$

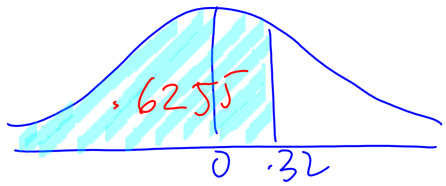
$= .5 + .1255 = .6255$





http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/Table-A4.pdf

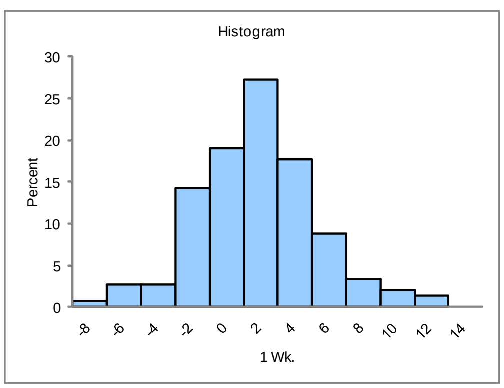
Ex. $Pr(Z \le 0.32)$: Table A.4



Ex. S&P 500 Index
500 large cap US

http://www.standardandpoors.com/indices/sp-500/en/us/?indexId=spusa-500-usduf--p-us-l--

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/GICS_500_Scorecard-1.xlsx



$\mu = 2.765\%$

$\sigma = 3.444\%$

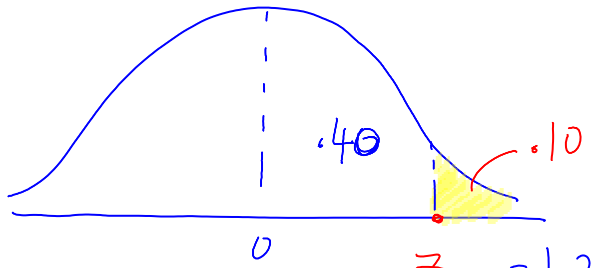
$$Pr(X \le 0) = Pr\left(\frac{X - 2.765}{3.444} \le \frac{0 - 2.765}{3.444}\right)$$

$$= Pr(Z \le -0.83) = 0.211$$

(ii) Calculation of z-value

Ex. Exam $\mu = 60$, $\sigma = 15$

Q: what is that score above which we have 10% of students?



$Z_{.10} = 1.28$

$$Z = \frac{X - \mu}{\sigma}$$

$$X = \mu + Z_{.10} \cdot \sigma = 60 + (1.28)15$$

$$= 79.2$$