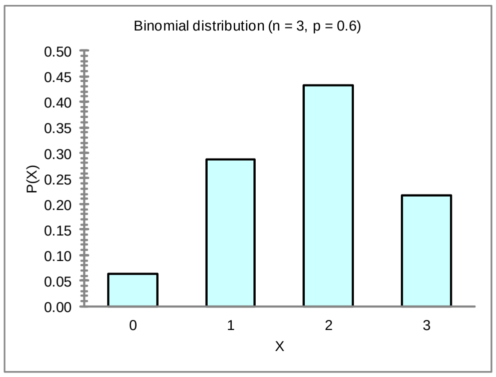
Binomial $\quad n \quad x$

p

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, \ldots, n$$

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/3Balls.xls



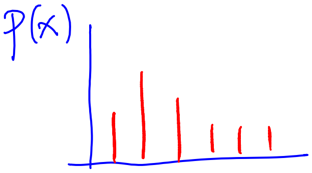
Ex. New drug adoption

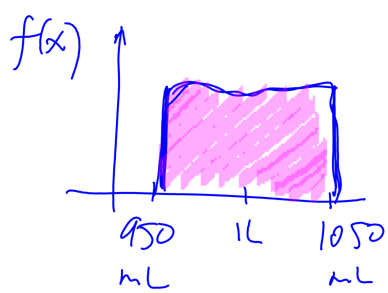
Ch. 5 Continuous Random Variables

(a) Continuous distributions

Rand. Var

Discrete $\qquad$ Continuous





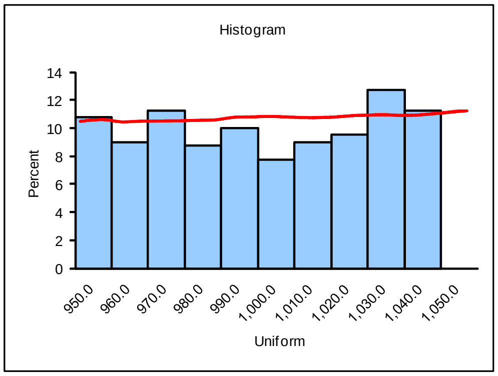
Sum of all heights = 1

Area = 1

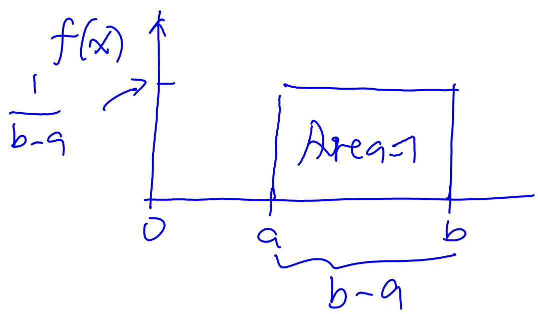
b) Uniform distribution

http://www.youtube.com/watch?v=dn1mIZS6I6o&feature=related

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/AppleJuice.xls



X uniform takes values between a and b



$(b-a)\cdot\frac{1}{b-a}=1$

Mean $\mu_X = \frac{1}{2}(a+b)$

s.d. $\sigma_X = \frac{b-a}{\sqrt{12}}$

$\sigma_X^2 = \sum (x-\mu_x)^2 p(x)$

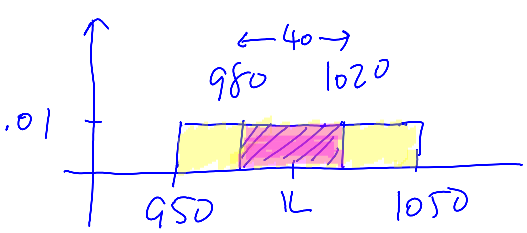
$\sigma_X^2 = \int_a^b (x-\mu_x)^2 \frac{1}{b-a}\,dx$

not too important

Ex. Apple juice

$a = 950$
$b = 1050$

$\frac{1}{b-a} = \frac{1}{100} = .01$



$Pr(980 \le X \le 1020) =$ Area of ■ region

$= 40 \times .01 = .40$

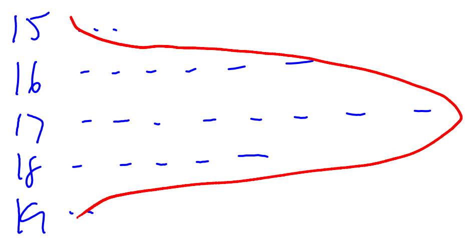
$Pr(X = 990) = \quad = 0$

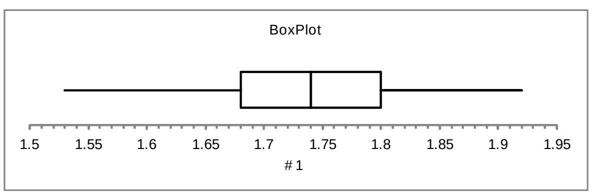
c) Normal distribution

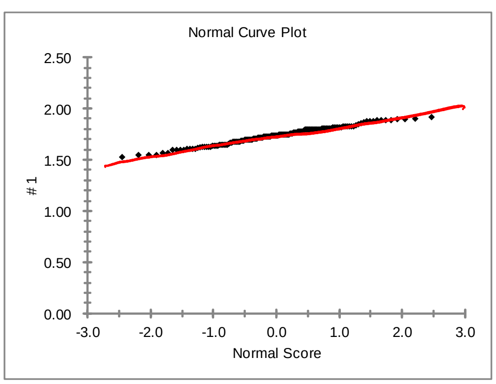
Ex. Heights data

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/Q600-2013-Scanned-Height-Gender-Handspan.pdf

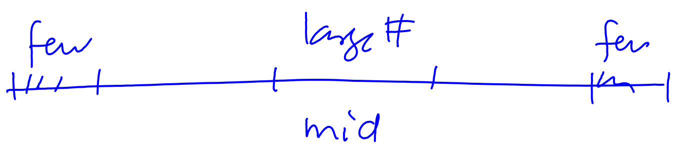
Stem/leaf



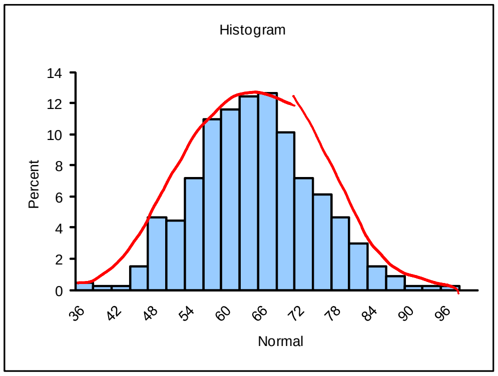




Ex. Test Scores in a large class



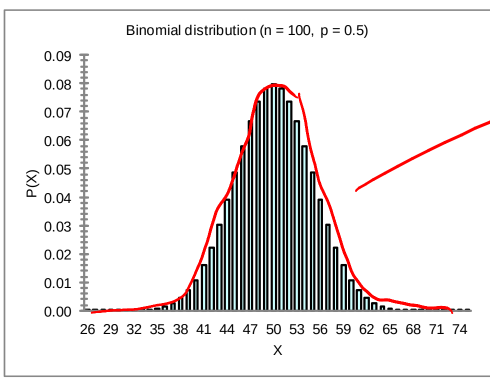
http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/NormalData-475_000.xls



Ex. Binomial with large n & $p \simeq 0.5$

$\downarrow$ 100

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/Binomial-Normal.xls



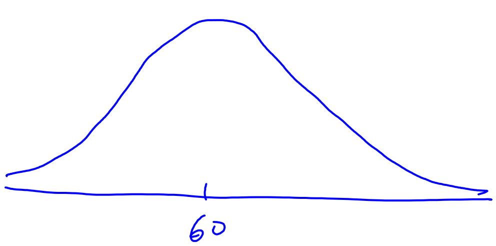
normal

Ex. Galton board

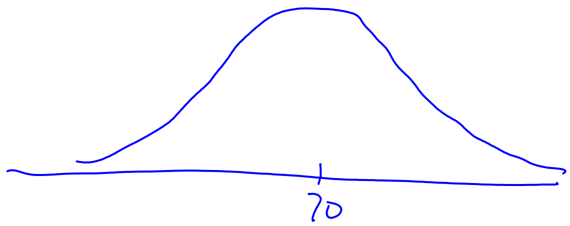
http://mathworld.wolfram.com/GaltonBoard.html

What's the effect of μ & σ on shape

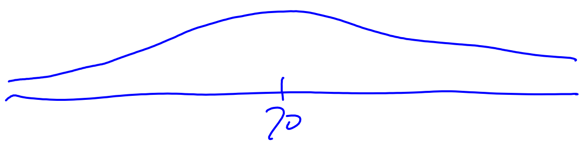
Ex.



$\mu = 60, \ \sigma = 3$



$\mu = 70, \ \sigma = 3$

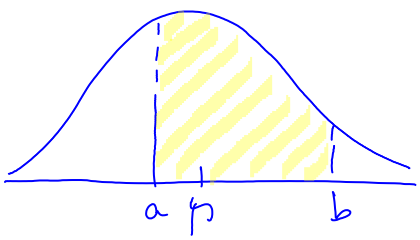


$\mu = 70, \ \sigma = 5$

μ: location
σ: dispersion

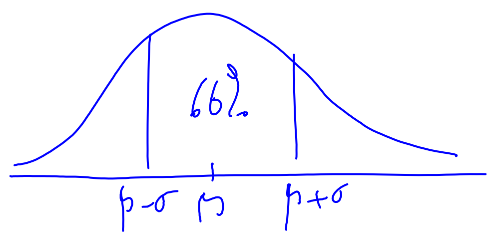
(i) Calculating probabilities

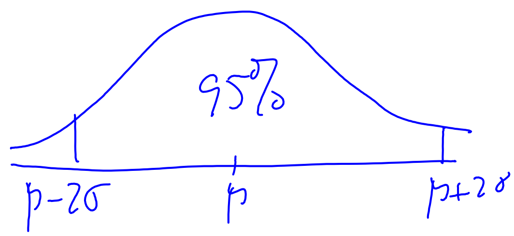
6

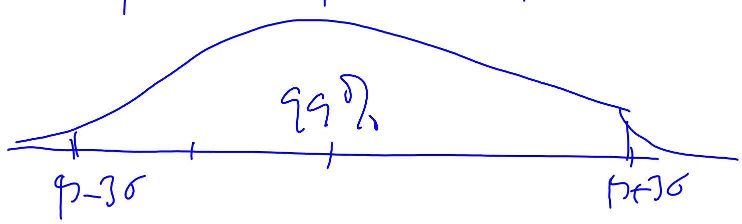


$Pr(a \le X \le b) = ?$

Empirical rule







How to find $Pr(a \le X \le b)$ for any (μ, σ)

Z-Scores: $Z > 0$: above mean by z std. dev

$Z < 0$: below " " " "

$$z = \frac{x-\mu}{\sigma}$$

X: normal (μ, σ)

Define a new r.v. Z as

$$Z = \frac{X-\mu}{\sigma}$$

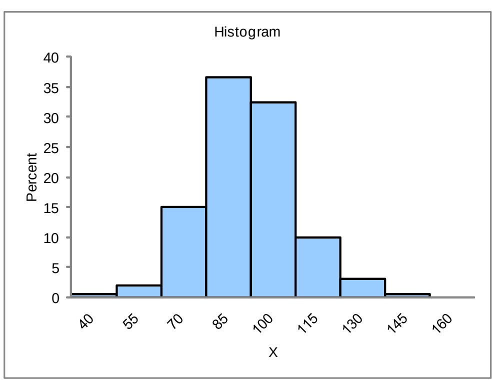
Most important result in Ch. 5

X normal with mean μ and std. dev. σ

Z " " " 0 " " " 1

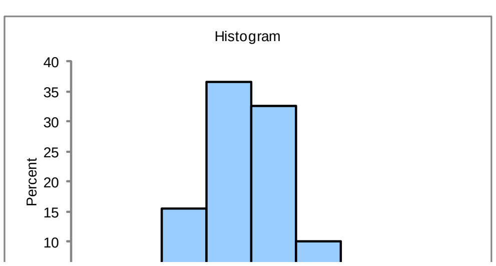
Ex. IQ

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/IQ-Scores-200-2009.xlsx

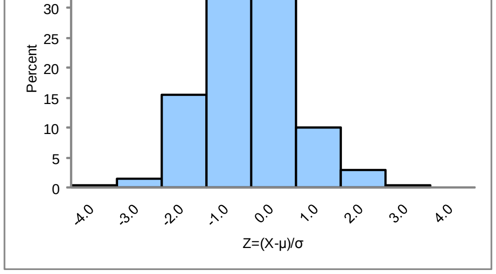


	X
count	200
mean	99.0955
sample variance	233.0254
sample standard deviation	15.2652
minimum	47.65
maximum	148.69
range	101.04

Pasted from <file:///C:\DOCUME~1\parlar\LOCALS~1\Temp\IQ-Scores-200-2009.xlsx>

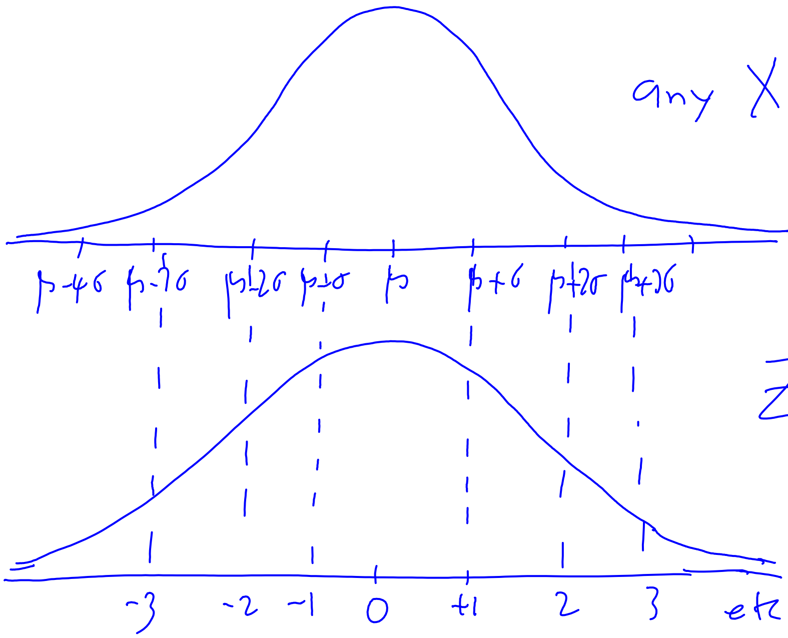


	Z=(X-μ)/σ
count	200
mean	-0.0603
sample variance	1.0357
sample standard deviation	1.0177
minimum	-3.49
maximum	3.246
range	6.736



Pasted from <file:///C:\DOCUME~1\parlar\LOCALS~1\Temp\IQ-Scores-200-2009.xlsx>

This Z is called standard normal r.v.



any X with (μ, σ)

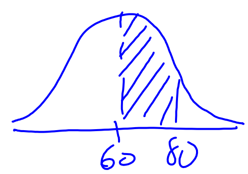
Z with $(0,1)$

$$z = \frac{x-\mu}{\sigma}$$

Ex. Test scores are normal, i.e.,

X normal with $\mu = 60$, $\sigma = 15$

What's $\Pr(60 \le X \le 80) = ?$



Standardize

$$60 \le X \le 80$$

$$\frac{60-60}{15} \le \underbrace{\frac{X-60}{15}}_{} \le \frac{80-60}{15}$$

$$\Downarrow$$

$$0 \le Z \le 1.33$$

find $\Pr(0 \le Z \le 1.33)$ (from table)

h3

$\sigma = 15$

h3

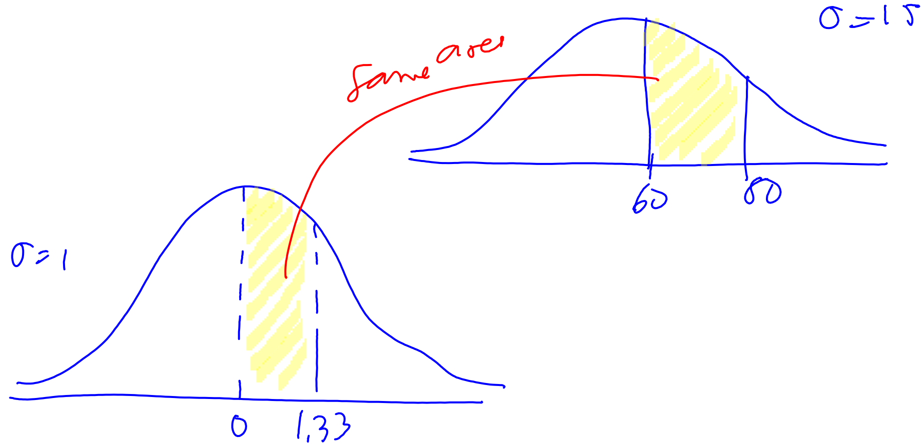
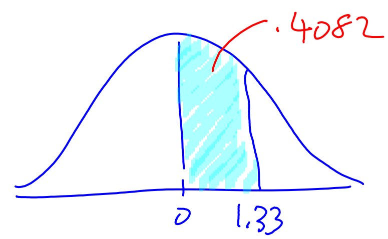


Table 5.1 on p.156 gives areas from 0 to z

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/NormalTable.wmf

Z	0.00	0.01	0.02	0.03	. . .	0.09
0.0						
0.1						
⋮						
1.0						
1.1						
1.2						
1.3				.4082		
⋮						



$Pr(0 \le Z \le 1.33) = 0.4082$

$Pr(60 \le X \le 80) = 0.4082$

What about $Pr(50 \le X \le 90) = ?$

$$50 \le X \le 90$$

$$\frac{50-60}{15} \le \underbrace{\frac{X-60}{15}}_{Z} \le \frac{90-60}{15}$$

$$\Pr(-.67 \le Z \le 2) =$$

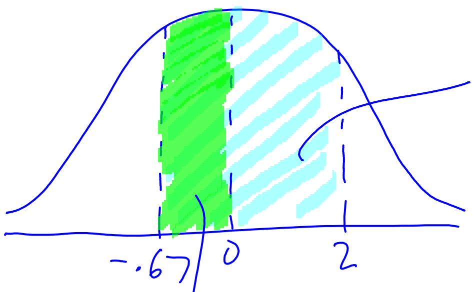
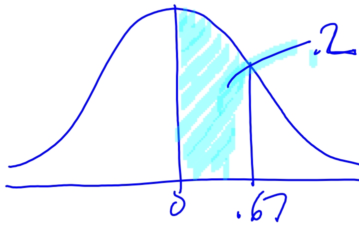


Table: .4772

same as the area from 0 to +.67 (Symmetry)



.2482

So total is

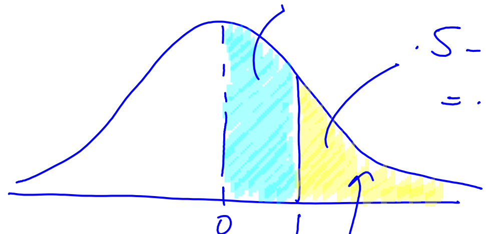
$$\begin{array}{r} .2486 \\ .4772 \\ \hline .7258 \end{array} = \Pr(50 \le X \le 90)$$

What about these?

.3413

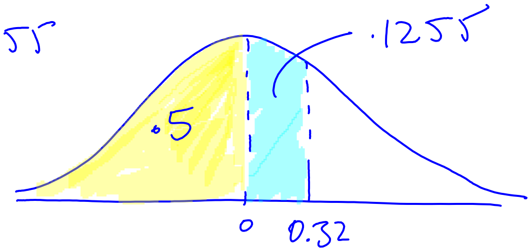
$.5 - .3413$
$= .1587$

$$\Pr(Z > 1) =$$



$$\Pr(Z \le 0.32) = .5 + .1255$$
$$= .6255$$

.1255

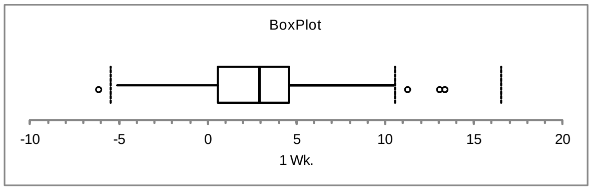


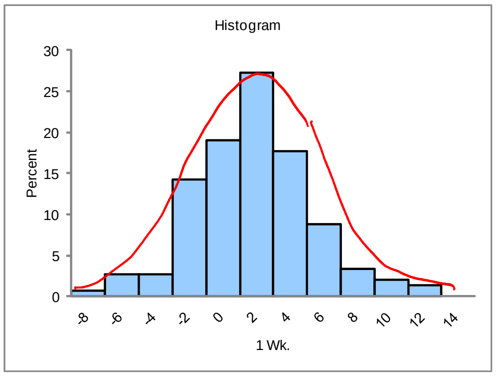
http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/Table-A4.pdf

Ex. S&P 500 index

http://www.standardandpoors.com/indices/sp-500/en/us/?indexId=spusa-500-usduf--p-us-l--

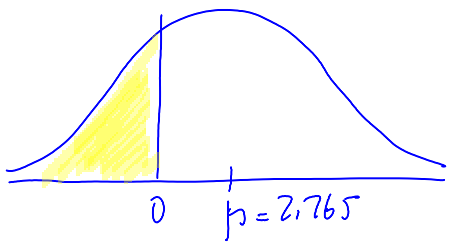
http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/GICS_500_Scorecard-1.xlsx





$\mu = 2.765\%$

$\sigma = 3.444\%$



$$\Pr(X \le 0) = \Pr\left(\frac{X-\mu}{\sigma} \le \frac{0-\mu}{\sigma}\right)$$

$$= \Pr\left(Z \le \frac{0-2.765}{3.444}\right)$$

$$= \Pr(Z \le -0.803)$$

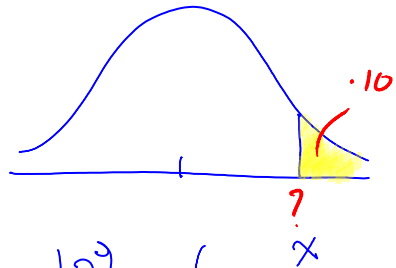
$$= 0.2119$$

Big table

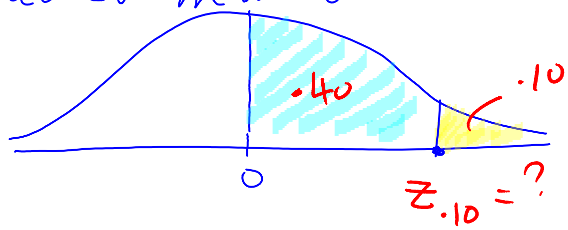
z	0.00	0.01
⋮		
$-0.8 \rightarrow$	0.2119	

(ii) Calculating z-values

Ex. Exam $\mu = 60$, $\sigma = 15$



Q: Above what score we have 10% of student marks?



z	0.00	. .	0.08
0.0			
⋮			
1.2			.3997

$z_{.10} = 1.28$

$z = \dfrac{x - \mu}{\sigma}$

$\Rightarrow \quad x = \mu + z_{.10}\sigma$

$= 60 + (1.28) \cdot 15$

$= 79.2$