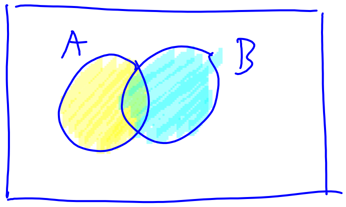
2013-10-02-C01

Wednesday, October 02, 2013
1:25 PM

(2) & (3) Union & Intersection

A

B



intersection

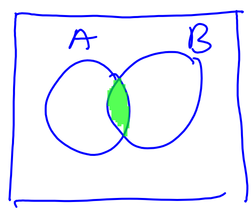
$$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$$

Ex. $A = \{11, 22, 33, 44, 55, 66\}$ same on both

$B = \{46, 55, 64\}$ sum = 10

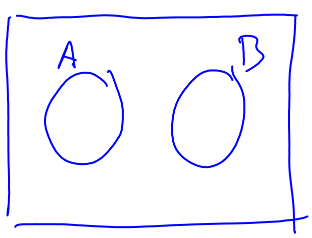
$$A \cup B = \{11, 22, 33, 44, 55, 66, 46, 64\}$$

$$A \cap B = \{55\}$$



(4) $Pr(S) = 1$

(5) Mutually exclusive events



A : H

B : T

$Pr(A \cap B) = 0$

d) Joint, marginal & conditional prob's

Ex. Product preference

1000 Surveyed

	Preference Coke (C)	Pepsi (P)	Total
Male (M)	200	300	500

Gender	Male (M)	200	300	500
	Female (F)	100	400	500
		300	700	1,000

Joint

$$Pr(M \cap C) = \frac{200}{1000} = .2$$

$$Pr(F \cap P) = \frac{400}{1000} = .4$$

etc

Marginal

$$Pr(C) = \frac{300}{1000} = .3$$

$$Pr(M) = \frac{500}{1000} = .5$$

etc

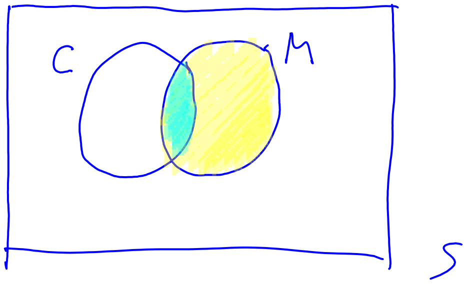
Summary

	C	P	Marginal
M	.2	.3	.5
F	.1	.4	.5
Marginal →	.3	.7	

Conditional

$Pr(C \overset{\text{given}}{|} M) = Pr(\text{prefers Coke given male})$

Venn



$$Pr(C|M) = \frac{Pr(C \cap M)}{Pr(M)} = \frac{.2}{.5} = .4$$

$$Pr(P|M) = \qquad = .6$$

$$Pr(C|F) = \qquad = .2$$

$$Pr(P|F) = \qquad = .8$$

		C	P	Total
given	M	.4	.6	1.0
	F	.2	.8	

e) Independence of events

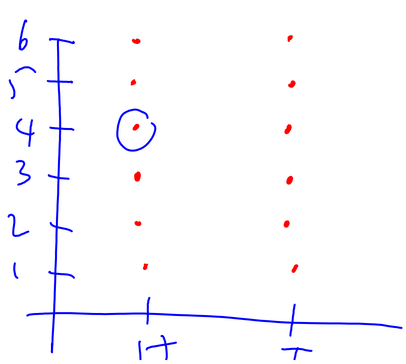
A and B are independent if

$$Pr(A|B) = Pr(A)$$

Ex.

A : die (4)

B : coin (H)



$Pr(A|B) = \frac{1}{6}$

$Pr(A) = \frac{1}{6}$

Note: $$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)}$$

$$\Rightarrow \quad Pr(A \cap B) = \underbrace{Pr(A|B)}_{Pr(A)} \cdot Pr(B)$$

$$Pr(4 \cap H) = Pr(4) \cdot Pr(H) = \frac{1}{6} \cdot \frac{1}{2} = \frac{1}{12}$$

Ex. Product preference

$Pr(C|M) = .4$, $Pr(C) = .3$

Ex. Lady Gaga on iPod

100 songs → shuffle

L: Lady Gaga on 1st song

$Pr(L) = \frac{1}{100}$, $Pr(\bar{L}) = \frac{99}{100}$

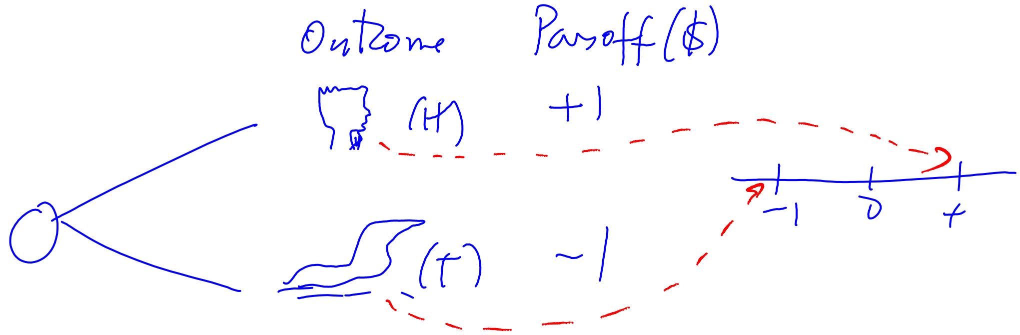
Three shuffles

$$Pr(\text{at least one } L) = 1 - Pr(\text{no } L)$$

$$= 1 - \frac{99}{100} \cdot \frac{99}{100} \cdot \frac{99}{100} \cong .03$$

Ch. 4. Discrete random variables

a) What is a random variable?



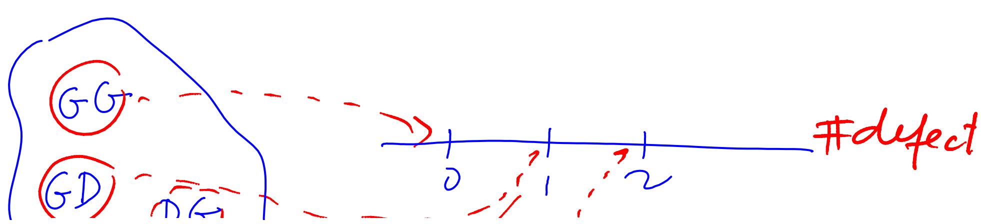
Def. A <u>random variable</u> associates a numerical value with each outcome of an experiment

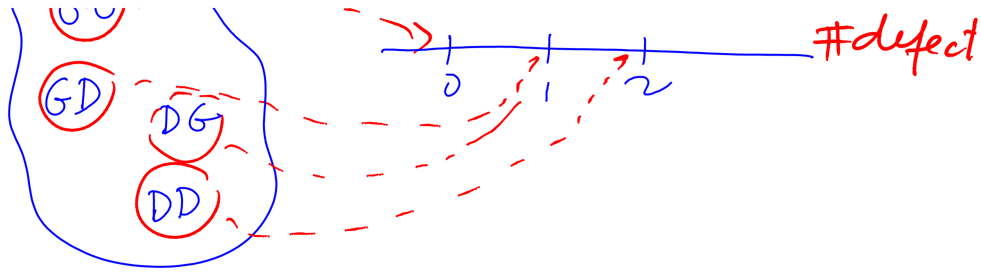
X : \$ payoff as a result of tossing coin
(discrete)
-1, +1

Ex. Mold tires in pairs
Interested in # of defective tires

#1	#2	Total def
G	G	0
G	D	1
D	G	1
D	D	2

X: # defectives in a pair   0, 1 or 2



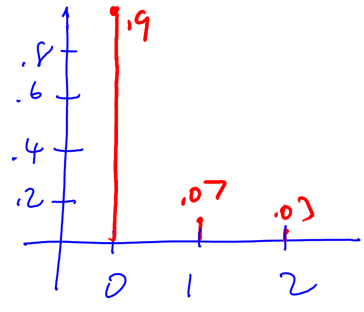


b) Discrete prob. distribution

$Pr(0 \text{ defect}) = 0.90 = p(0)$

$Pr(1 \quad '' \quad) = 0.07 = p(1)$

$Pr(2 \quad '' \quad) = 0.03 = p(2)$



x	p(x)
0	.90
1	.07
2	.03

prob. distr

100 runs (200 tires)

		Total def
Approx.	90 out of 100 have 0 defects	0
	7 out of 100 " 1 "	7
	3 " " " 2 "	6
		13

Avg. # defects/run $= \frac{13}{100} = .13$

" " " /tire $= \frac{13}{200} = .065$

Ex. Bicycle sales at Pierik's

Last 100 business days

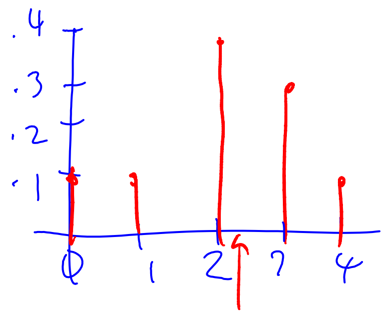
# sold (x)	# days	$p(x)$	Total sales
0	10	.1	0
1	10	.1	10
2	40	.4	80
3	30	.3	90
4	10	.1	40
	100	1.0	220

Average sales / day $= \frac{220}{100} = 2.2$

Mean (expected) value of X

$$E(X) = \mu_X = \sum_{\text{all } x} x\,p(x) \qquad \left(\mu = \frac{1}{N}\sum x\right)$$

x	$p(x)$	$x\,p(x)$
0	.1	0
1	.1	.1
2	.4	.8
3	.3	.9
4	.1	.4
		2.2



Ex. Home insurance

House worth \$500,000
Premium 400/yr

Outcomes	Profit (x)	Prob
Fire	$400 - 500{,}000 = -499{,}600$	.0001
No fire	400	.9999

X: company profit

$$E(X) = \mu_X = .0001(-499{,}600) + .9999(400)$$

$$E(X) = \mu_X = .0001(-499{,}600) + .9999(400)$$
$$= \$350$$

(would you insure if only 1 house?)

$$10{,}000 \times 400 = 4{,}000{,}000$$
$$-\ 500{,}000$$
$$3{,}500{,}000 \div 10{,}000 = \$350$$

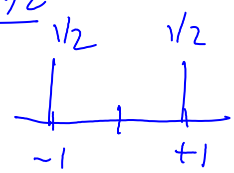
Recall $\quad \sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2$

Rand. var $\quad \sigma_X^2 = \sum (x - \mu_X)^2 p(x)$

$$\sigma_X = \sqrt{\sigma_X^2}$$

Ex. Coin flip

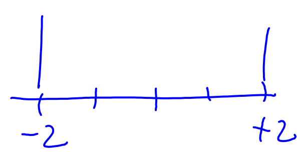
Outcome	x	$p(x)$	$(x-\mu_X)^2$	$(x-\mu_X)^2 p(x)$
H	+1	1/2	1	1/2
T	−1	1/2	1	1/2
				$\Sigma = 1$



$\mu_X = +1\left(\frac{1}{2}\right) + (-1)\frac{1}{2} = 0$

$\sigma_X^2 = 1$

$\sigma_X = 1$



Ex. Higher risk/reward

Outcome	x	$p(x)$	$(x-\mu_X)^2$	$(x-\mu_X)^2 p(x)$
H	+2	1/2	4	2
T	−2	1/2	4	2
				$\sigma_X^2 = 4$

$\ddot{T}$ $\dot{-}2$ $\dot{1}/\bar{2}$ $\dot{4}$ $\overline{2}$

$\sigma_x^2 = 4$

$\mu_x = 0$

$\sigma_x^2 = 4$

$\sigma_x = 2$

Ex. Loaded coin (H always)

Outcome	x	$p(x)$	$(x-\mu_x)^2$	$(x-\mu_x)^2 p(x)$
H	+1	1	0	0
T	-1	0	4	0
				$\sigma_x^2 = 0$

$\mu_x = +1(1) + (-1) \cdot 0 = 1$

c) Binomial distribution

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/3Balls.xlsh
ttp://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/Binomial.pd
f