2013-09-25-C01

Wednesday, September 25, 2013
1:36 PM

$$x = \mu + z\sigma$$

$$\frac{x-\mu}{\sigma} = z$$

Ex. Test Scores in a small class

x: 56, 72, 83, 92, 100

$\mu = 80.60$
$\sigma = 15.44$

x	$x-\mu$	$z=(x-\mu)/\sigma$
56	-24.6	-1.6
72	-8.6	-0.6
83	2.4	0.2
92	11.4	0.7
100	19.4	1.3

Ex. Quality improvement (Coffee temp)

Accept range

67 71 75

(71 ∓ 4)

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/CoffeeTemp-Tolerance.xls

73
76
69
67
74
70
69
72
71
68
75
72
67
74
72
68
70
77
68
72
69
75
68
73

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count	24
mean	71.21
sample variance	8.87
sample standard deviation	2.98
minimum	67
maximum	77
range	10

$\bar{x} = 71.21$
$s = 2.98$

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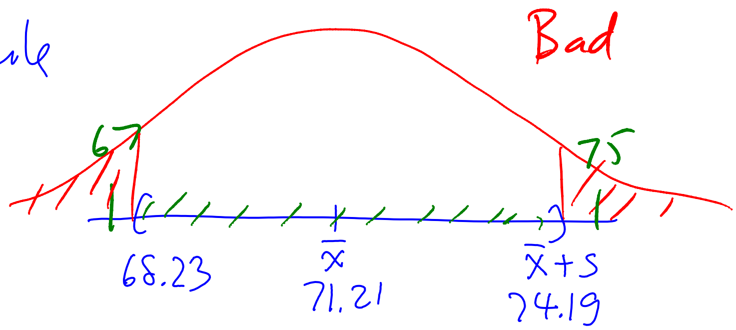
Empirical rule

~68%

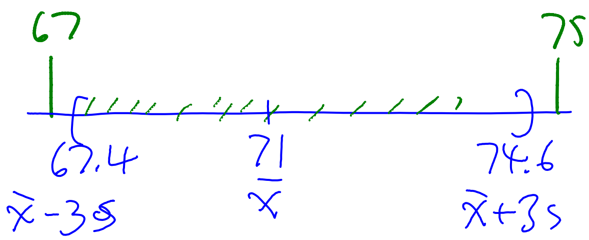


Adjust & take a new sample

We find $\bar{x} = 71$

$s = 1.2$

99% in $(71 \mp 3(1.2)) = [67.4, 74.6]$



Good!

d) More measures of variation

Ex. GMAT

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/GMAT-Percentiles.pdf

http://www.testmasters.net/GmatAbout/Scoring-Scale

The **pth percentile** of a group of *n* measurements is a value such that (approximately) *p*% of measurements fall at or below the value and (approximately) (100-*p*)% fall at or above the value.

Pasted from <http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/ch-02.html>

Md = 50th percentile

Ex. Exam scores in a small class

$n=8$

Scores	36	40	72	74	74	80	86	90
Posit.	1	2	3	4	5	6	7	8

56 74 83

Position of $p=25$ 2.5

" $p=50$ 4.5

" $p=75$ 6.5

One leans (36)

14.28% 28.51% (72) (74)

$n=7$							
Scores	40	72	74	74	80	86	90
Pos.	1	2	3	4	5	6	7

Pos. of
$p=25$ — 2
$p=50$ — 4
$p=75$

An easy method for locating the position of the *p*th percentile of *n* measurements:

- First, order all measurements and calculate *i = (p/100)*n*.
- If *i* is an integer, then the position is the average of measurements in positions *i* and *i+1*.
- If *i* is not an integer, then the position is the next integer greater than *i*.

Pasted from <http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/ch-02.html>

$$i = \frac{p}{100}\cdot n$$

$n=7$

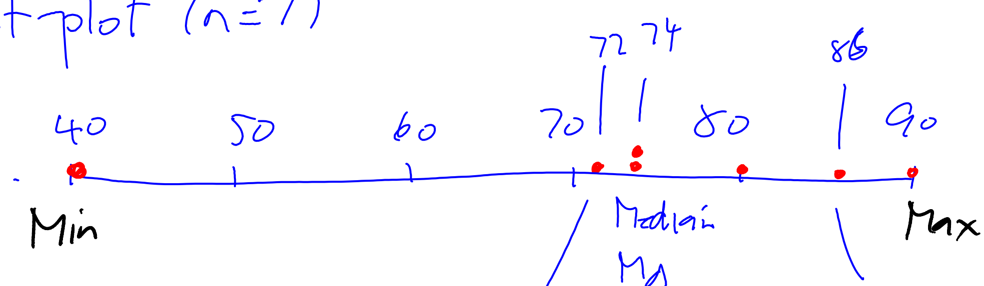
$p=25$, $\quad i=\frac{25}{100}\cdot 7 = 1.75 \longrightarrow 2 \longrightarrow 72$

$p=75$, $\quad i=\frac{75}{100}\cdot 7 = 5.25 \longrightarrow 6 \longrightarrow 86$

$n=8$

$p=25$, $\quad i=\frac{25}{100}\cdot 8 = 2 \rightarrow$ position $=\frac{1}{2}(2+3)=2.5 \rightarrow 56$

Dot-plot ($n=7$)



Q_1 1st quartile $\qquad$ Q_3 3rd quartile

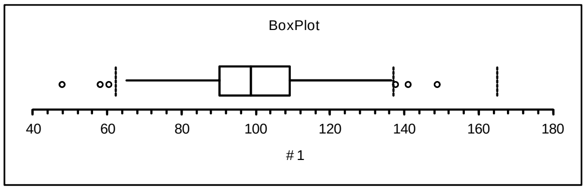
Q_1 $\qquad$ Q_3

Interquartile range: $Q_3 - Q_1 = 14$

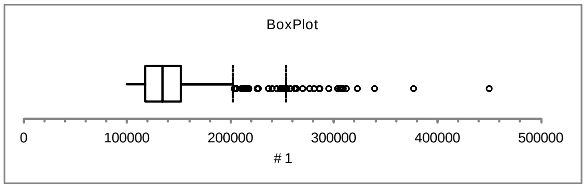
Box-whisker plot

Box-whisker plot

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/Box-Whisker.pdf



IQ



Mac Salaries

Ch. 3 Probability

Ex Coin toss

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/CoinToss.xls

Ex. Lotto 6/49

"Lotto 2/4"

1	2	3	4
1	2		
1		3	
1			4
	2	3	
	2		4
		3	4

$Pr(win) = \frac{1}{6}$

Ex. Birthday

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/2.DB-2013-C01.pdf

http://en.wikipedia.org/wiki/Birthday_paradox

Ex. Monty Hall's "Let's Make a Deal"

Decision: Choose D1 & switch

	Car behind	Monty will open	Switch
D1 — $\frac{1}{3}$	D1	D2 or D3	Goat
$\frac{1}{3}$	D2	D3	Car
$\frac{1}{3}$	D3	D2	Car

$\left.\begin{matrix}\text{Car}\\ \text{Car}\end{matrix}\right\} 2/3$

Methods

- Classical
  - equally likely outcomes
  - $P(H) = Pr(T) = \frac{1}{2}$
- Empirical (freq)
- Subjective

Random experiment: uncertain outcomes

Ex. Coin toss

Outcomes: H, T

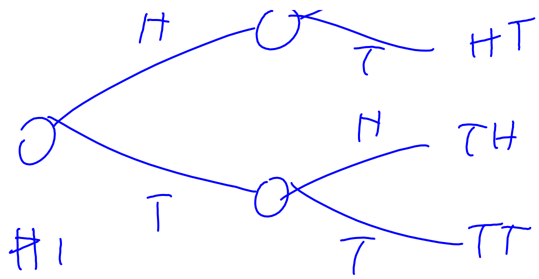
Ex. Two coins

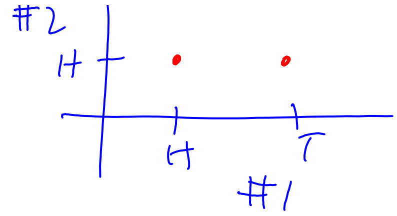
H — H: HH

H — T: HT

T

#2





5) Sample spaces & events

$S =$ set of all outcomes of a random experiment

Ex. Coin $S = \{H, T\}$

Ex. Two coins $S = \{HH, HT, TH, TT\}$

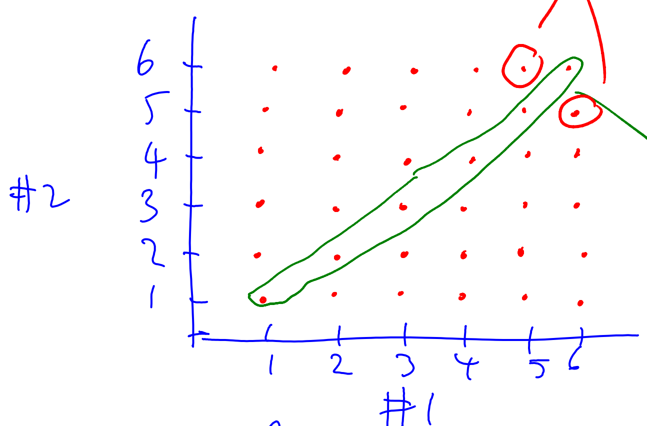
$$\Pr(HH) = \frac{1}{4}$$

Ex. Backgammon

http://en.wikipedia.org/wiki/Backgammon

Sum=11 → $\{56, 65\}$

$S = \{11, 12, \ldots, 66\}$



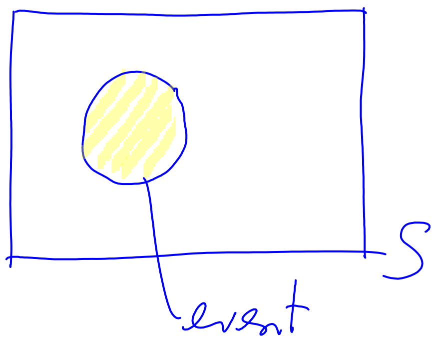
Same on both $= \{11, 22, 33, 44, 55, 66\}$

$$\Pr\{\overbrace{\text{same on both}}^{A}\} = \frac{6}{36} = \frac{1}{6} = 0.167$$

$$\Pr\{\underbrace{\text{sum} = 11}_{B}\} = \frac{2}{36} = \frac{1}{18} = 0.055$$

$$\Pr\{A \text{ or } B\} = 0.167 + 0.055 = 0.222$$

An <u>event</u> is a collection of outcomes



Venn diagram

Ex. Lotto 6/49

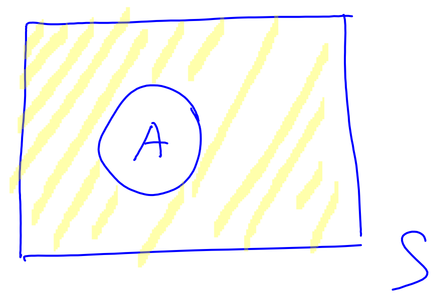
$$\Pr(\text{win}) = \frac{\text{your 6 \#s}}{\text{all possible combi's}} = \frac{1}{13{,}983{,}816} = .0000715$$

c) Some rules to calculate prob's

(i) Complement

A : some event

$\bar{A}$: Complement of A



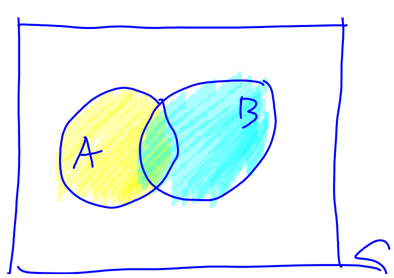
Ex. $A = \{HH, TT\}$ $\quad \Pr(A) = \frac{1}{2}$

$\bar{A} = \{HT, TH\}$ $\quad \Pr(\bar{A}) = 1 - \Pr(A) = \frac{1}{2}$

(2) Union

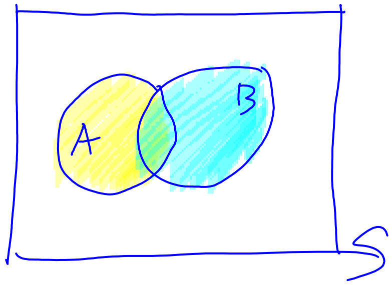
A : event

B : event



A: event

B: event



$A \cup B$: union

Ex. Two dice

$A = \{11, 22, 33, 44, 55, 66\}$ same on both

$B = \{46, 55, 64\}$ sum = 10

$A \cup B = \{11, 22, 33, 44, 55, 66, 46, 64\}$

$$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(\text{green})$$

$Pr(A \cap B)$