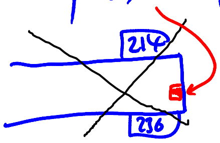
2011-11-07-EC01

Monday, November 07, 2011
6:41 PM

HW (21)

HW4 due Nov. ~~14~~, ~~dropbox in front of Jeanette's office~~

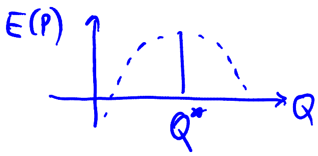
In class



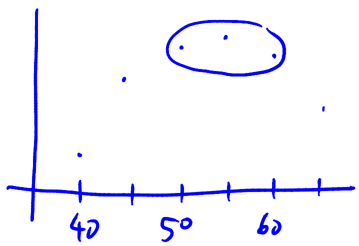
Exam Nov. 11, 7-10

Ex. Newsboy (Cont'd)

Profit = Sales Rev − Purch. cost + Salv. val



Q	E(P)
40	40
45	44.16
50	46.64
55	47.62
60	46.46
65	43.97



2. Generating random numbers

Excel =RAND( )

Computer uses a recursive method

RN(n+1) = function [RN(n)]

2.1 Mixed congruential method

$$x_{i+1} = (a x_i + c) \quad \text{modulo } m$$

$i = 0, 1, 2.$

x_0 : given seed value

$$RN \Rightarrow \quad R_{i+1} = \frac{x_{i+1} + \frac{1}{2}}{m}, \quad i = 0, 1, 2, -$$

14 mod 4 = 2

x_{i+1} is the remainder after $ax_i + c$ is divided by m

Ex. $a = 5$, $c = 7$, $m = 8$, $\boxed{x_0 = 4}$

27 mod 8 = 3

i	x_i	$ax_i + c$	x_{i+1}	R_{i+1}
0 →	4	27	3	.4375
1	3	22	6	.8125
2	6	37	5	.6875
3	5	32	0	.0625
⋮	⋮	⋮	.	⋮

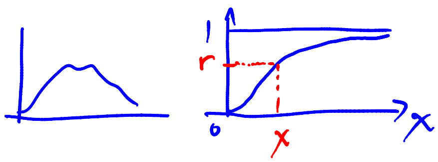
MCG.xls			Mixed congruential			

			generator			
a =	5		x[i+1] = ax[i] +c, mod m;			i = 0,1,2,...
c =	7		R[i+1] = (x[i+1]+ 1/2)/m,		i = 0,1,2,...	
m =	8					
x[0] =	4					
i	x[i]	ax[i]+c	x[i+1]	R[i+1]		
0	4	27	3	**0.4375**		
1	3	22	6	**0.8125**		
2	6	37	5	**0.6875**		
3	5	32	0	**0.0625**		
4	0	7	7	**0.9375**		
5	7	42	2	**0.3125**		
6	2	17	1	**0.1875**	If the parameters are not chosen carefully,	
7	1	12	4	**0.5625**	cycling may occur.	
8	4	27	3	**0.4375**	<- Cycling starts here	
9	3	22	6	**0.8125**		
10	6	37	5	**0.6875**		
11	5	32	0	**0.0625**		
12	0	7	7	**0.9375**		
13	7	42	2	**0.3125**		
14	2	17	1	**0.1875**		
15	1	12	4	**0.5625**		
16	4	27	3	**0.4375**		
17	3	22	6	**0.8125**		
18	6	37	5	**0.6875**		
19	5	32	0	**0.0625**		
20	0	7	7	**0.9375**		
21	7	42	2	**0.3125**		
22	2	17	1	**0.1875**		
23	1	12	4	**0.5625**		
24	4	27	3	**0.4375**		
Best values (for a 32-bit machine) are						
m = 2^31-1	:	2,147,483,647				
a = 7^5	:	16807				
c = 0						

Pasted from <file:///C:\DOCUME~1\parlar\LOCALS~1\Temp\MixedCongruentialGenerator.xls>

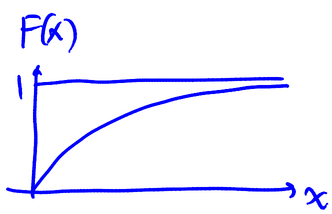
1.2 Inverse tranformation

X $\quad f(x)$ $\quad F(x)$



$r = F(x)$

$x = F^{-1}(r)$



Ex. X exp: $\quad f(x) = \lambda e^{-\lambda x}$

$F(x) = 1 - e^{-\lambda x}$

$r = \overbrace{F(x) = 1 - e^{-\lambda x}}$

$e^{-\lambda x} = 1 - r$

$\ln(e^{-\lambda x}) = \ln(1-r)$

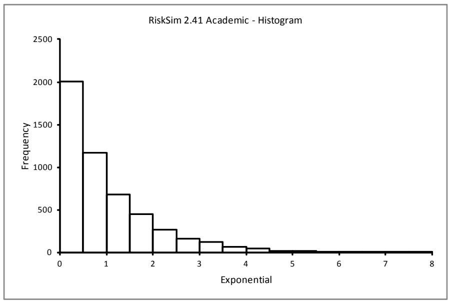
$-\lambda x = \ln(1-r)$

$x = -\frac{1}{\lambda}\ln(1-r) \quad \longleftarrow \quad x = F^{-1}(r)$

If $r = .85$ and $\lambda = 1$ $\rightarrow$

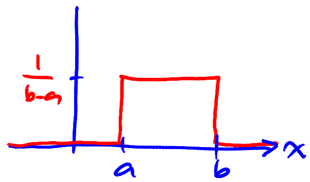
If $r = .89$ and $\lambda = 1 \Rightarrow$

$$x = -\frac{1}{1}\ln(1-.89) = 2.21$$

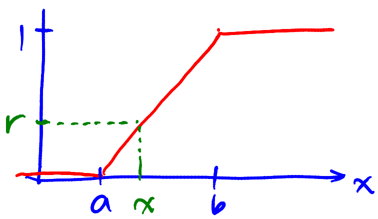


Ex. Uniform

$$f(x) = \frac{1}{b-a}, \quad a \le x \le b$$



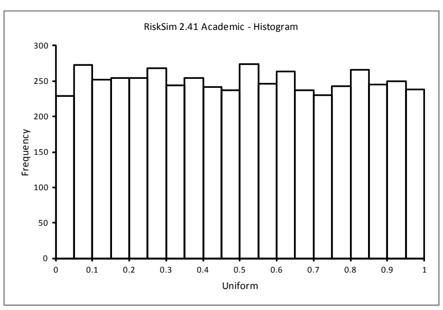
$$F(x) = \frac{x-a}{b-a}$$



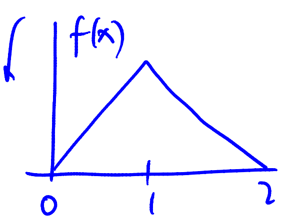
$$r = \frac{x-a}{b-a} = F(x)$$

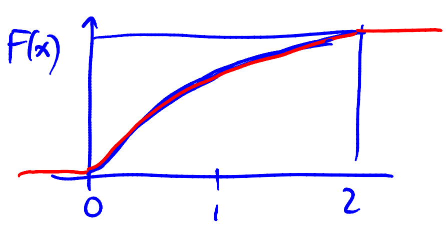
$$\Rightarrow \quad x = a + (b-a)\, r$$

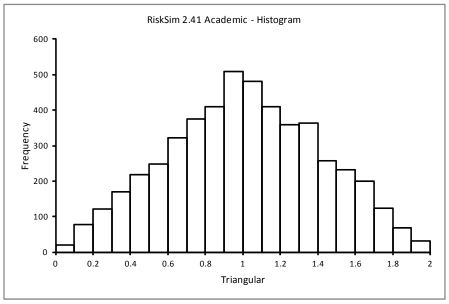
If $a=3$, $b=5$, $r=.2 \Rightarrow x = 3.4$



Ex. Triangular







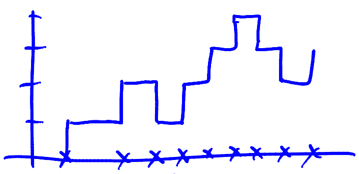
$$f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2-x, & 1 \le x \le 2 \end{cases}$$

$$F(x) = \begin{cases} x^2/2, & 0 \le x \le 1 \\ -1+2x-\frac{1}{2}x^2, & 1 \le x \le 2 \end{cases}$$

Because Farhad asked

3. Discrete event simulation: (b), (d), (e)

Ex. G/G/1 Queue



event change the state: arrival / departure

Arrival: Server idle → enters service
" busy → join queue

Departure: Queue empty → server idle
" ~~not~~ " → serve customer in queue

Next event time advance

Interarrival times (min)	Prob	Service times (min)	Prob
IT: 1	.15	ST: 1	.3
2	.20	2	.4
3	.40	3	

IT times (min)	Prob
1	.15
2	.20
3	.40
4	.15
5	.10

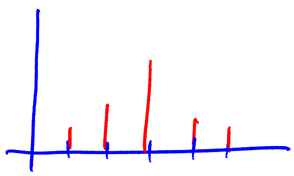
$\frac{1}{\lambda} = E(IT) = 2.85$ min/cust

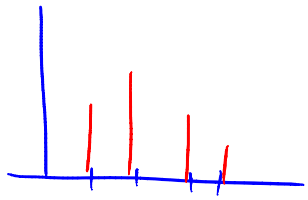
$\lambda = \frac{1}{2.85} = .35$ cust/min

ST times (min)	Prob
1	.3
2	.4
3	.2
4	.1

$\frac{1}{\mu} = E(ST) = 2.1$ min/cust

$\mu = \frac{1}{2.1} = .47$ cust/min





not expon!

Assume #1 arrives at TM=0 and simulate for 12 min

Cust. #	Arrival Time (3)	IT (1)	ST (2)
1	0	1	2
2	1	2	2
3	3	2	3
4	5	1	1
5	6	5	2
6	11		1

$\underbrace{\text{IT} \quad \text{ST}}_{\text{generated}}$

Simulated $E(ST) = \frac{11}{6} = 1.84$

http://www.business.mcmaster.ca/courses/O711/ChapterComments/documents/12minSimulation.pdf

$Est(P_0), \ldots$

$$Est(L) = \sum_{n=0}^{3} n P_n = 0\frac{1}{12} + 1\cdot\frac{6}{12} + 2\frac{4}{12} + 3\frac{1}{12} = 1.41$$

$$Est(L_q) = \sum_{n=1}^{2} (n-1) P_n = \qquad = 0.5$$

Show the following

$$Est(W) = \frac{17}{6} = 2.83$$

$$Est(W_q) = \frac{6}{6} = 1$$

Queueing Simulator

We can simulate more general queues using the Excel file <Queueing_Simulator.xls>. In order for this to work, you should do the following: Excel Options > Trust Center > Trust Center Settings > Macro Settings > Enable All Macros.

Pasted from <http://www.business.mcmaster.ca/courses/O711/ChapterComments/Ch-20-HL.html>

We will first discuss a simple case of the M/M/1 queue (Innis Library case) with mean $1/\lambda = 1/100$ for interarrivals, and mean $1/\mu = 1/120$ for service time. The simulated results will agree what we found from our formulas. Here's the Excel file. The theoretical results for this problem were L = 5, Lq = 4.17, W= 3 min (0.05 hr) and Wq = 2.5 min (0.04 hr).
We will also discuss an example with G/E(4)/5 where G is Uniform(0,6) and $1/\mu = 10$ for E(4), i.e., k = 4. This would be impossible to analyze using any formulas.

Pasted from <http://www.business.mcmaster.ca/courses/O711/ChapterComments/Ch-20-HL.html>

Ch.15 Decision Analysis

Ex. Oil drilling

Ex. New product

uncertainty ⟷ sequential decisions

a) Basic concepts

Ex. Sunspotz Co. solar heating units

Should capacity be ↑ ?

Decisions (alternatives): A_1: expand $5M

A_2: modernize $2M

States of nature S_1: high demand (0.4)

S_2: normal " (0.6)

		Demand	Cost	Income	Net
$\boxed{A_1}$ expand	s_1	high	-5	10	5
	s_2	normal	-5	4	-1
$\boxed{A_2}$ modernize	s_1	High	-2	5	3
	s_2	normal	-2	2.5	0.5

Payoff table

		High S_1	Normal S_2
Exp	A_1	5	-1

Payoff table

		High S_1	Normal S_2
Exp	A_1	5	-1
Mod	A_2	3	0.5
	Prior pr.	.4	.6

b) Decision making w/o experimentation

Maximin

	S_1	S_2	Min
A_1	5	-1	-1
A_2	3	.5	.5 ← max
	.4	.6	

Maximum likelihood

	S_1	S_2
A_1	5	-1
A_2	3	.5
	.4	.6

↑ max likelihood

Bayes's rule (expected monetary value – EMV)

$E(A_1) = .4(5) + .6(-1) = 1.4$

$E(A_2) = .4(3) + .6(0.5) = 1.5 \leftarrow$

EMV = 1.5

Sensitivity analysis

$p = Pr(S_1)$

	S_1	S_2
A_1	5	-1
A_2	3	.5
	p	$1-p$

$p \quad 1-p$

$E(A_1) = 5p + (-1)(1-p) = 6p - 1$

$E(A_2) = 3p + .5(1-p) = 2.5p + 0.5$