2011-09-12-EC01
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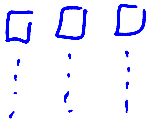
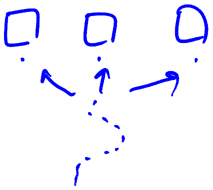
Ch.24 Review of Probability

Ex. Uncertainty

Finance: Stock prices, etc
Prod'n: Demand
Marketing: Market share
Stores: 3 servers



vs



a) Basic Probability

Sample space + events

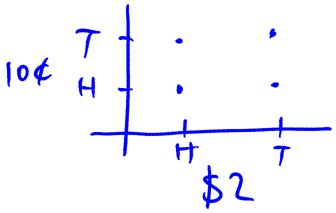
Ex. Flip coin $S=\{H,T\}$

sample space

outcomes

Ex. Toss a die $S=\{1,2,\dots,6\}$

Ex. Two coins



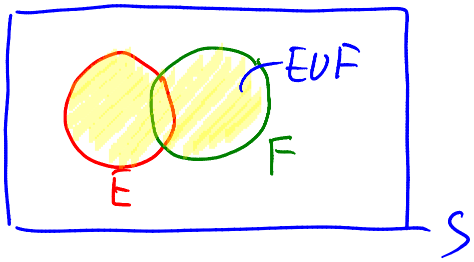
Ex. Gasoline Sold

$$S=\{x \mid 0\le x<\infty\}$$

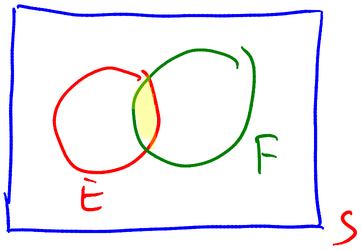
Event E: Any collection of outcomes

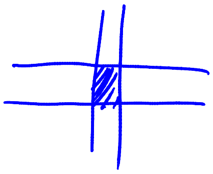
Ex. Die $E=\{\text{even \#s}\}=\{2,4,6\}$

$E \cup F$: union of events E and F

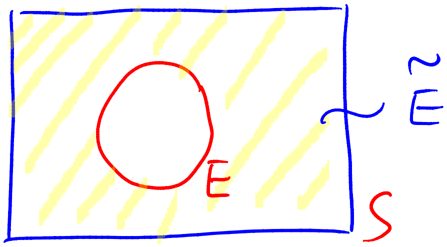


$E \cap F$: intersection





$\tilde{E}$: complement of E (not in E)



Pr(E): probability of E

Rules

① $0 \leq Pr(E) \leq 1$

② $Pr(S) = 1$

③ If E_1 and E_2 are mutually exclusive, then

$$Pr(E_1 \cup E_2) = Pr(E_1) + Pr(E_2)$$

④ $Pr(\tilde{E}) = 1 - Pr(E)$

Ex. Die $E_1 = \{1,2\}$

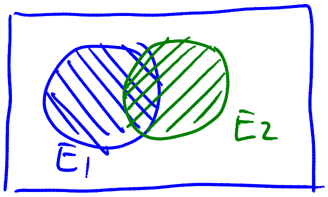
$E_2 = \{4,5,6\}$

$$Pr(E_1 \cup E_2) = Pr(E_1) + Pr(E_2)$$
$$= \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$$

$$Pr(E) = \frac{\#\text{ events in } E}{\#\text{ events in } S}$$

⑤ If E_1 + E_2 are <u>not</u> mutually exclusive,

$$Pr(E_1 \cup E_2) = Pr(E_1) + Pr(E_2) - Pr(E_1 \cap E_2)$$



Ex. Test scores + eventual positions in a Company

≥70 H
<70 L

Senior S
Junior J
Dismissed D

1000

Freq Table

	S	J	D	
H	67	400	200	667
L	33	167	133	333
	100	567	333	1000

Prob

$Pr(H \cap S)$

	S	J	D	
H	.067	.400	.200	.667 = Pr(H)
L	.033	.167	.133	.333

← marginal

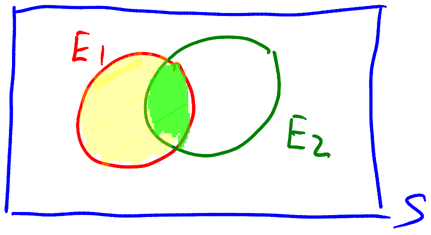
	.100	.567	.333	1.000

$$Pr(H \cup J) = Pr(H) + Pr(J) - Pr(H \cap J)$$

$$= \underbrace{.667 + .567}_{1.234} - .4 = .834$$

Conditional Prob's

$$Pr(E_2 | E_1) = \frac{Pr(E_1 \cap E_2)}{Pr(E_1)}$$



Ex. Employ't

$$Pr(D | H) = \frac{Pr(D \cap H)}{Pr(H)} = \frac{200/1000}{667/1000} = \frac{.200}{.667} = .299$$

<u>Independence</u> Two events A & B are <u>ind't</u> if

$$Pr(A \cap B) = P(A) \cdot Pr(B)$$

So, if A & B are ind't, then

$$Pr(A | B) = \frac{Pr(A \cap B)}{Pr(B)} = \frac{Pr(A)\cancel{Pr(B)}}{\cancel{Pr(B)}} = Pr(A)$$

∴ B has no relevance

Ex. Employment

$Pr(H\cap J) = 0.400$ ← $\neq$ → not ind't

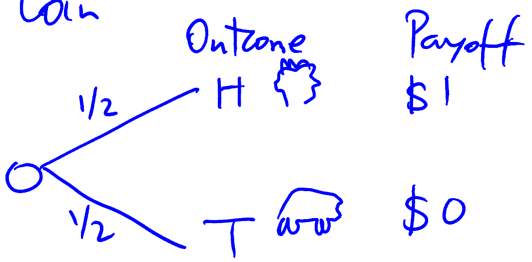
$Pr(H)\cdot Pr(J) = 0.667 \times .567 = .378$

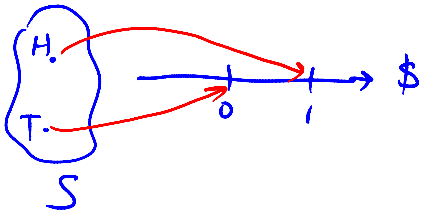
$Pr(S\mid H) = .10$	$Pr(S) = .10$	Ind't
$Pr(J\mid H) = .60$	$Pr(J) = .567$	dep
$Pr(D\mid H) = .30$	$Pr(D) = .333$	

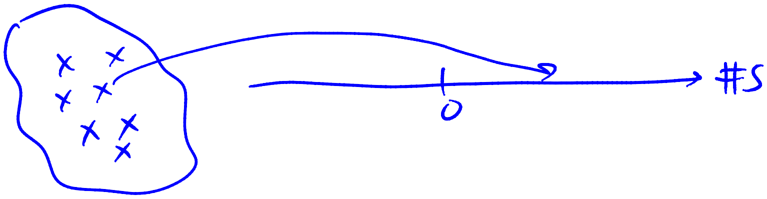
b) Random Variabls

Associate a nbr. with an outcome

Ex. Coin





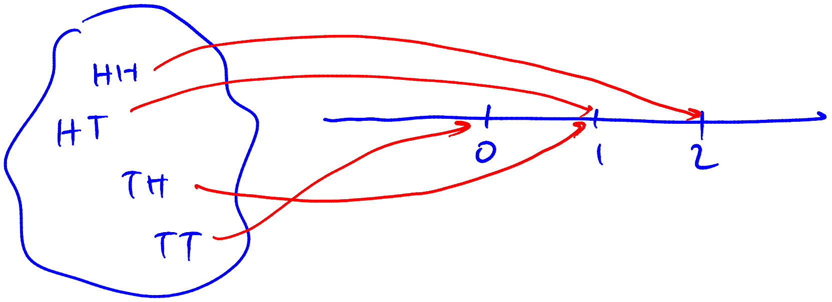


b.1. Discrete r.v's

Ex. Toss 2 fair coins $S = \{\underbrace{HH}_{2}, \underbrace{HT}_{1}, \underbrace{TH}_{1}, \underbrace{TT}_{0}\}$

X : #heads appearing

random var.

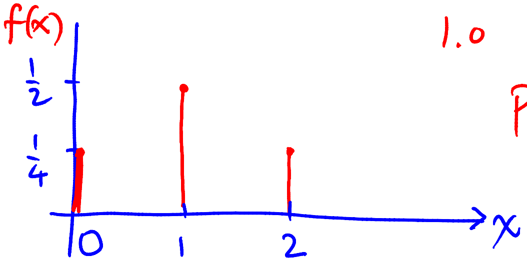


$Pr(X=0) = Pr(T,T) = \frac{1}{4}$ ✓

$Pr(X=1) = Pr(H,T) + Pr(T,H) = \frac{1}{2}$ ✓

$Pr(X=2) = Pr(H,H) = \frac{1}{4}$ ✓

1.0



Prob. mass function (pmf)

$f(x) = Pr(X=x)$

"Sum of all spikes" = 1

Cumul. distrib. funct (cdf)

	pmf	cdf
x	f(x)	F(x)
0	1/4	1/4
1	1/2	$1/4 + \frac{1}{2} = \frac{3}{4}$
2	1/4	1