Transient Sol'n of a Finite State CTMC using LT.

Define $\tilde{P}_{ij}(s) = \int_0^\infty e^{-st} P_{ij}(t)\,dt \quad , \; i,j = 1, \ldots, N < \infty$

$$\tilde{P}(s) = [\tilde{P}_{ij}(s)] \quad \text{Matrix}$$

Consider $\quad P'(t) = Q P(t), \quad P(0) = I$

Taking LT, $\quad s\tilde{P}(s) - P(0) = Q\tilde{P}(s)$

$$s\tilde{P}(s) - I = Q\tilde{P}(s)$$

$$\Rightarrow \quad \tilde{P}(s) = (sI - Q)^{-1}$$

Ex. Two State CTMC $\qquad 0 \underset{\mu}{\overset{\lambda}{\rightleftarrows}} 1$

$$Q = \begin{bmatrix} -\lambda & \lambda \\ \mu & -\mu \end{bmatrix}$$

$$\tilde{P}(s) = (sI - Q)^{-1} = \frac{1}{s(s+\lambda+\mu)} \begin{bmatrix} s+\mu & \lambda \\ \mu & s+\lambda \end{bmatrix}$$

$\rho = \lambda + \mu$

$$= \frac{1}{s} \begin{bmatrix} \frac{\mu}{\lambda+\mu} & \frac{\lambda}{\lambda+\mu} \\ \frac{\mu}{\lambda+\mu} & \frac{\lambda}{\lambda+\mu} \end{bmatrix} + \frac{1}{s+\lambda+\mu} \begin{bmatrix} \frac{\lambda}{\lambda+\mu} & -\frac{\lambda}{\lambda+\mu} \\ -\frac{\mu}{\lambda+\mu} & \frac{\mu}{\lambda+\mu} \end{bmatrix}$$

$$\Rightarrow \quad P(t) = \begin{bmatrix} \frac{\mu}{\rho} & \frac{\lambda}{\rho} \\ \frac{\mu}{\rho} & \frac{\lambda}{\rho} \end{bmatrix} + e^{-\rho t} \begin{bmatrix} \frac{\lambda}{\rho} & -\frac{\lambda}{\rho} \\ -\frac{\mu}{\rho} & \frac{\mu}{\rho} \end{bmatrix}$$