2. Continuous r.v.'s

X, Y have joint density $f(x,y)$, assume $x \geq 0, y \geq 0$

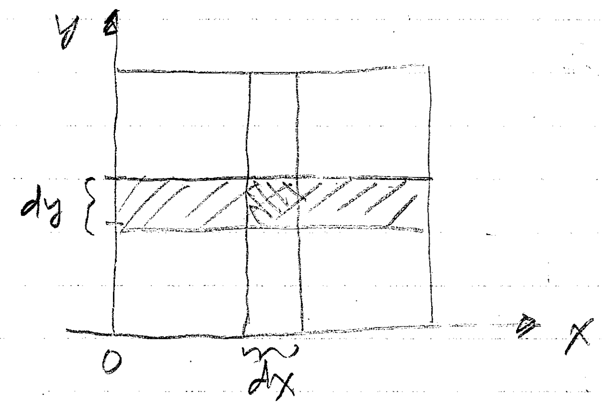
$$f_{X|Y}(x|y) = \frac{f(x,y)}{f_Y(y)} \quad \text{where } f_Y(y) = \int_0^\infty f(x,y)\,dx$$

similar to $p_Y(y) = \sum_x p(x,y)$

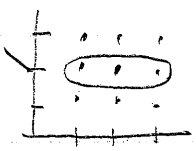
<u>Visualization</u>

$$f_{X|Y}(x,y)\,dx = \frac{f(x,y)\,dx\,dy}{f_Y(y)\,dy}$$

$$\frac{\text{Volume over } ▩}{\text{Volume over } ▨}$$

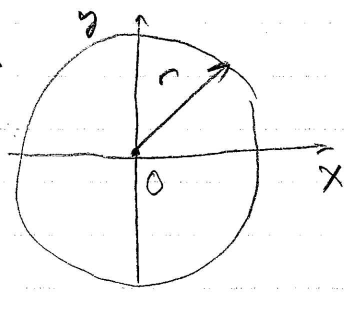


Similar to discrete



Ex. Throwing darts $\quad f(x,y) = \frac{1}{\text{circle area}} = \frac{1}{\pi r^2}, \quad x^2+y^2 \leq r^2$

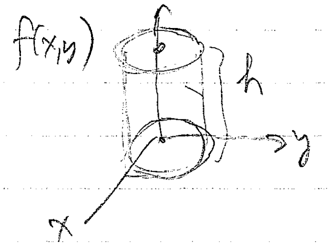
Always in circle, uniformly



$$\iint_{x^2+y^2 \leq r^2} f(x,y)\,dx\,dy = 1$$

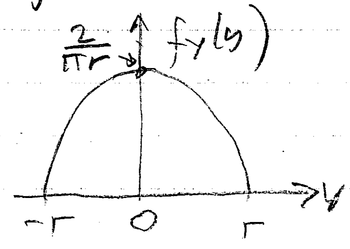
$$f_Y(y) = \int_{x^2+y^2 \leq r^2} \frac{1}{\pi r^2}\,dx = \frac{1}{\pi r^2} \int_{-\sqrt{r^2-y^2}}^{\sqrt{r^2-y^2}} dx$$

$$= \frac{2}{\pi r^2}\sqrt{r^2-y^2}, \quad |y| \leq r$$



Vol $= \pi r^2 h$

Not uniform



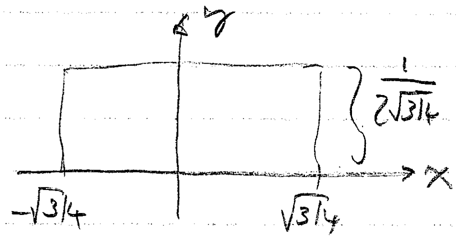
$$f_{X|Y}(x|y) = \frac{f(x,y)}{f_Y(y)} = \frac{\frac{1}{\pi r^2}}{\frac{2}{\pi r^2}\sqrt{r^2-y^2}} = \frac{1}{2\sqrt{r^2-y^2}}, \quad x^2+y^2 \le r^2$$

no x here!

For fixed y, $f_{X|Y}$ is uniform

for example, if $y = \frac{1}{2}$, $r = 1$,

then $f_{X|Y}(x|\tfrac{1}{2}) = \dfrac{1}{2\sqrt{1-\frac{1}{4}}} = \dfrac{1}{2\sqrt{3/4}}$, $\sqrt{\frac{3}{4}} \le x \le \sqrt{\frac{3}{4}}$



of course

$$\int_{x^2+y^2\le r^2} f_{X|Y}(x|y)\,dx = 1$$