20

9 16.41

~~7~~. Absorbing Chains

In some problems one or more states are absorbing, i.e., once process enters these, it can't leave

Ex. Accounts Receivable Analysis.

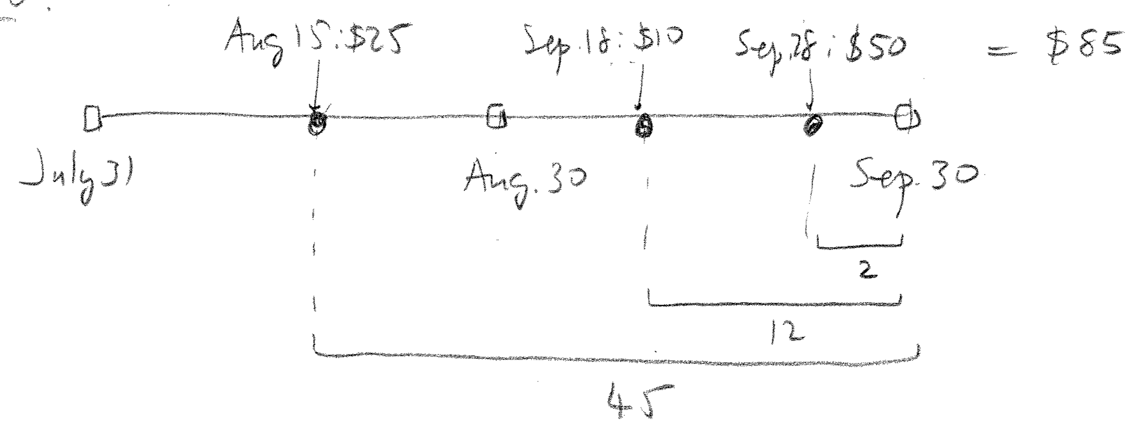
(real, -) (Robinson's) Large dept. store has two ageing categories for its accounts receivables

1. 0-30 days old
2. 31-90 " "

If any portion exceeds 90 days → Bad debt

Age is determined according to __oldest__ bill at end of day

__Sep. 30:__



45 days oldest → 31-90 day categ. for all $85

31-90 → 0-30 __Oct. 7:__ Paid Aug. 15 bill of $25

Remaining = $60

Category = 0-30 days since $60 is 19 days old

12+7=19

	Sep. 30	Oct. 7	Oct. 14	Oct. 21	Oct. 28	. . .
Total	$85 $\xrightarrow{-25}$	$60 $\xrightarrow{0}$	$60 $\xrightarrow{-10}$	$50 $\xrightarrow{-50}$	0	
Age	45	19	26	13	0	
Categ.	2	1	1	1	P	

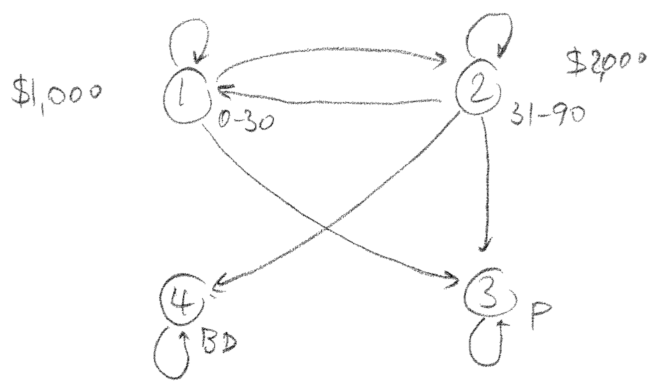
Suppose on Dec. 31, \$3,000 in accounts receivable

$1,000 (0-30 day) and $2000 (31-90 day)

Q: How much will eventually be collected?

States
1. 0-30
2. 31-90
3. Paid
4. Bad debt

} Weekly transitions



Process ends after a random # of transition

30% of 0-30 group stay in 0-30

$$P = \begin{array}{c|c} & \begin{array}{cccc} \text{0-30} & \text{31-90} & & \\ 1 & 2 & P & BD \end{array} \\ \begin{array}{c} \text{0-30 } 1 \\ \text{31-90 } 2 \\ P \\ BD \end{array} & \left[\begin{array}{cc|cc} .3 & .3 & .4 & 0 \\ .3 & .1 & .4 & .2 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \end{array} = \left[\begin{array}{c|c} Q & R \\ \hline 0 & I_a \end{array}\right]$$

Same dim as # of abs. states (2 here)

$$P^2 = \left[\begin{array}{c|c} Q^2 & R + QR \\ \hline 0 & I_a \end{array}\right]$$

$$P^3 = \left[\begin{array}{c|c} Q^3 & R + QR + Q^2R \\ \hline 0 & I_a \end{array}\right]$$

$$P^n = \left[\begin{array}{c|c} Q^n & N_n R \\ \hline 0 & I \end{array}\right] \to P^\infty = \left(\begin{array}{c|c} Q^\infty & NR \\ \hline 0 & I \end{array}\right)$$

16.43

In general,

$$P^n = \left[\begin{array}{c|c} Q^n & N_n R \\ \hline 0 & I_t \end{array}\right] \to P^\infty = \left(\begin{array}{c|c} Q^\infty & NR \\ \hline 0 & I_t \end{array}\right)$$

Same dim as transient state

where $N_n = I_t + Q + Q^2 + \cdots + Q^{n-1}$

$N = \lim_{n \to \infty} N_n = (I_t - Q)^{-1}$ (Note: $1 + x + x^2 + \cdots + \cdots = \frac{1}{1-x}$, $|x| < 1$)

Our example: $N = \begin{bmatrix} 1.67 & .56 \\ .56 & 1.30 \end{bmatrix}$

So, NR: prob. of ending up in an absorbing state given we start elsewhere

$$NR = \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cc} P & BD \end{array} \\ \begin{bmatrix} .89 & .11 \\ .74 & .26 \end{bmatrix} \end{array}$$

Suppose $B = [b_1, b_2] = [1000, 2000]$

$ in 0-30 on Dec.31 — $ in 31-90 on Dec.31

$$BNR = \underset{\text{0-30} \qquad \text{31-90}}{\begin{pmatrix} 1000 & 2000 \end{pmatrix}} \begin{array}{c} \\ \text{0-30} \\ \text{31-90} \end{array} \begin{array}{c} \begin{array}{cc} P & BD \end{array} \\ \begin{bmatrix} .89 & .11 \\ .74 & .26 \end{bmatrix} \end{array}$$

$$= [890 + 1480,\ 110 + 520] = \underset{P \qquad BD}{[2370,\ 630]}$$