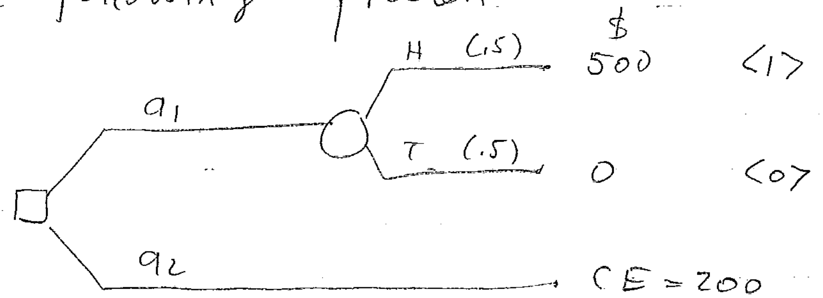
Risk Aversion

Consider the following problem

$EU(a_1) = .5$

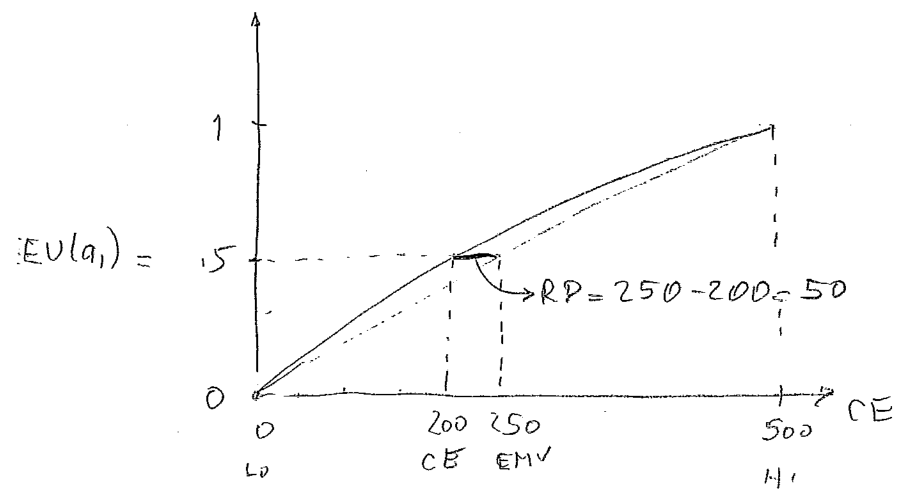


If your $CE = 200 < EMV(a_1) = \$250$ then you are unwilling to "play the odds". You are conservative or risk averse. You prefer certainty of, say \$205 to 50/50 chances of 0 and \$500.

In this case you are willing to forego an <u>expected</u> amount of \$50 <u>not</u> to play the gamble

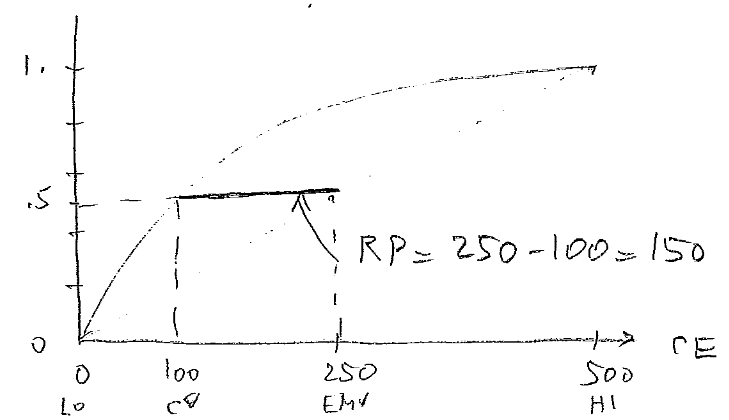
Risk Premium: $\boxed{RP = EMV - CE}$

$RP > 0 \Rightarrow$ Risk averse



From $EU(a_1)$ we can get CE using utility curve

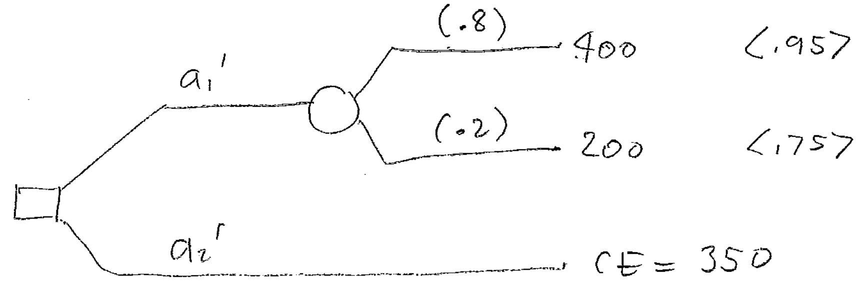
If more risk averse



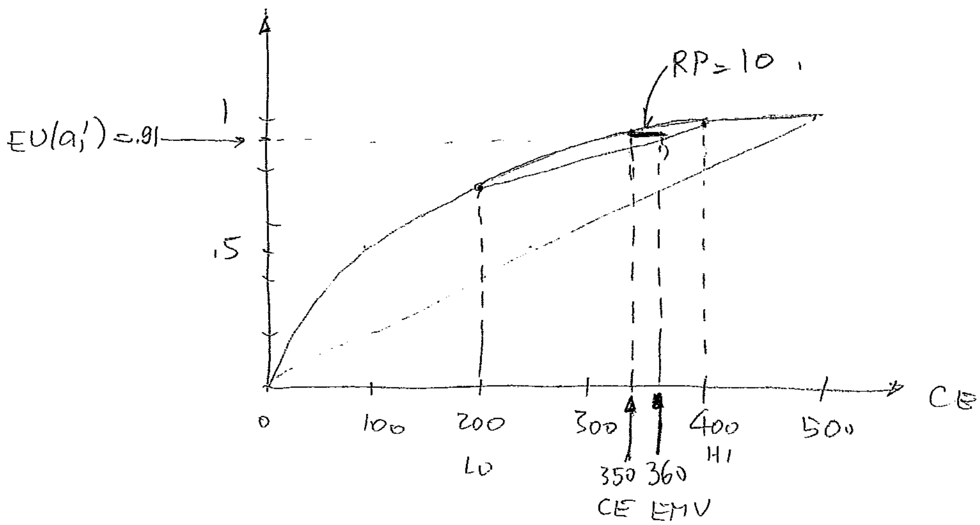
∴ Depending on the degree of risk aversion, RP differs.

RP also changes depending on the quantities involved

Ex.

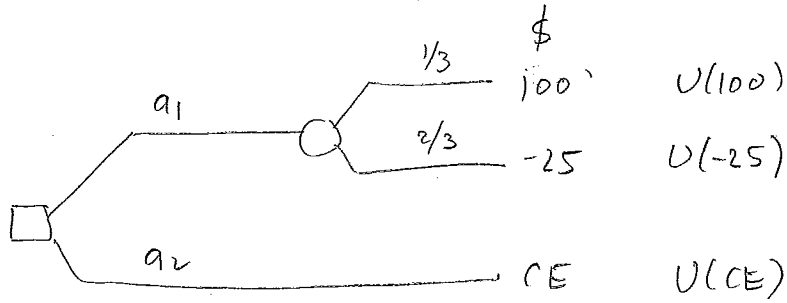


$EU(a_1') = .8(.95) + .2(.75) = .91$, $EMV(a_1') = .8(400) + .2(200) = 360$

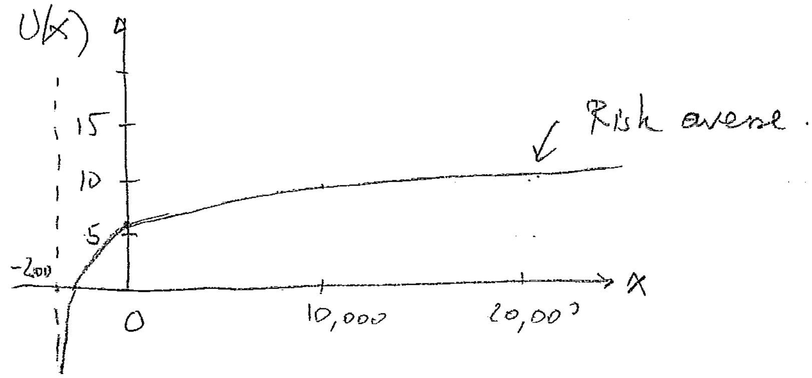


RP = 360 − 350 = 10

Ex.



What's the RP if $U(x) = \log_e(x+200)$?



$EMV(a_1) = \frac{1}{3}(100) + \frac{2}{3}(-25) = 16.67$

CE is a point x_c where

$$U(x_c) = EU(a_1)$$

$$\log_e(x_c + 200) = \frac{1}{3}U(100) + \frac{2}{3}U(-25)$$

$$= \frac{1}{3}\log_e(300) + \frac{2}{3}\log_e(175) \Rightarrow$$

$$x_c = 9.44$$

$\therefore \quad RP = 16.67 - 9.44 = \$7.23$

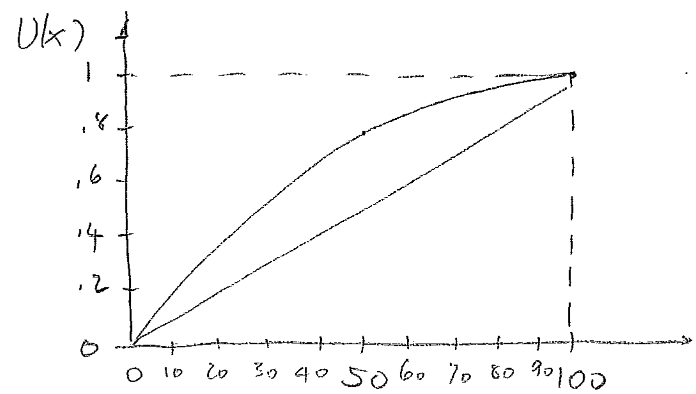
Problem: Show that above utility function $U(x) = \log_e(x+200)$ is always concave, i.e. $U''(x) < 0$.

Fact: Risk aversion $\Leftrightarrow$ concave utility function

Decreasing Risk Aversion

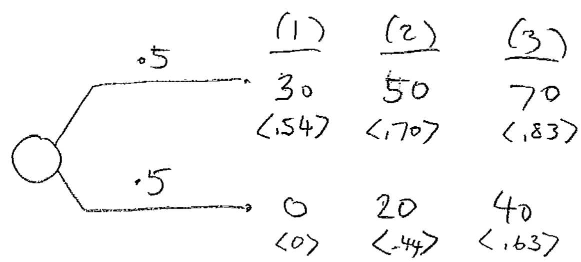
This implies that degree of risk aversion decreases as payoffs increase

Ex. Consider the utility curve $U(x) = .1\sqrt{x} = .1x^{\frac{1}{2}}$



Concave U(x).

Examine the following Three 50/50 gambles



Difference is same

50/50 Gamble	EMV	CE	RP	EU
0, 30	15	7.5	7.5	.27
20, 50	35	33.3	1.7 Decr.	.13
40, 70	55	53.9	1.1	.10

As min. payoff ↑, one becomes less risk averse.

Pratt's risk aversion function $r(x)$.

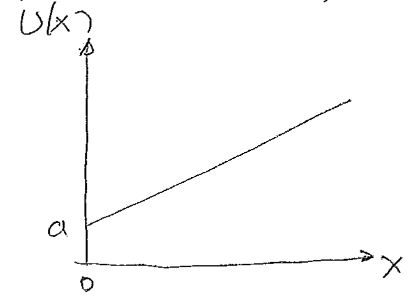
Let $U(x)$ be the utility function of a decision maker. Then

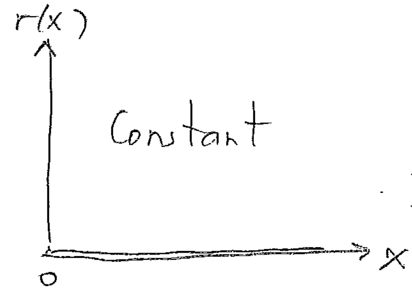
$$r(x) = -\frac{U''(x)}{U'(x)}$$

is known as Pratt's risk aversion function, (absolute)

Ex. $U(x) = a + bx$, $U'(x) = b$, $U''(x) = 0$

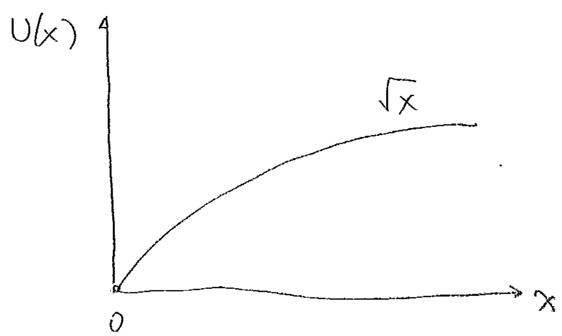
$$r(x) = -\frac{0}{b} = 0$$

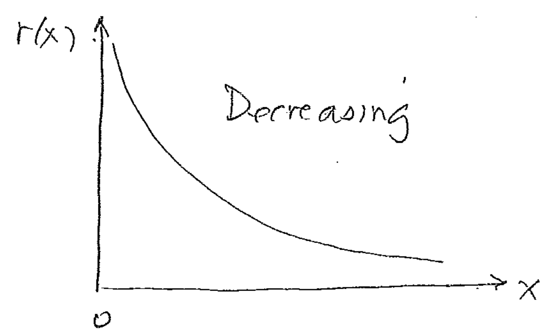




Ex. $U(x) = \sqrt{x} = x^{\frac{1}{2}}$, $U'(x) = \frac{1}{2}x^{-\frac{1}{2}}$, $U''(x) = -\frac{1}{4}x^{-\frac{3}{2}}$

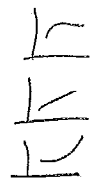
$$r(x) = -\frac{-\frac{1}{4x^{3/2}}}{\frac{1}{2x^{1/2}}} = \frac{1}{4x^{\frac{3}{2}}} \cdot 2x^{\frac{1}{2}} = \frac{1}{2}x^{\frac{1}{2}-\frac{3}{2}} = \frac{1}{2}x^{-1} = \frac{1}{2x}$$





Why is $r(x)$ useful?

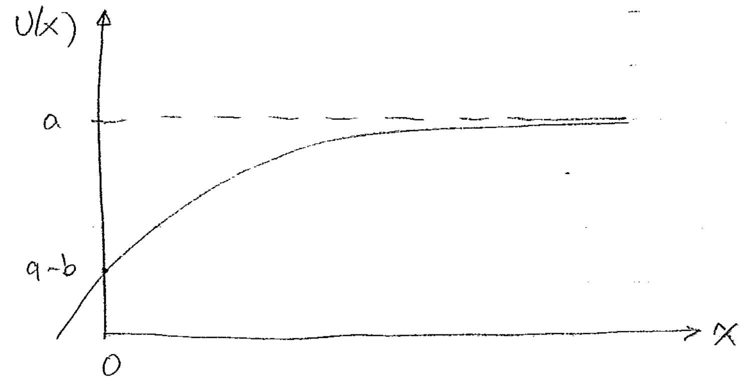
Concave functions $\Rightarrow$ $U''(x) < 0$
Linear functions $\Rightarrow$ $U''(x) = 0$
Convex " $\Rightarrow$ $U''(x) > 0$



Obviously U is always increasing $\therefore$ $U'(x) > 0$

$\therefore$ $r(x) > 0$ $\Rightarrow$ risk averse
$r(x) = 0$ $\Rightarrow$ risk neutral
$r(x) < 0$ $\Rightarrow$ risk prone

Ex. Let $U(x) = a - b e^{-rx}$, $a, b > 0$, $r > 0$

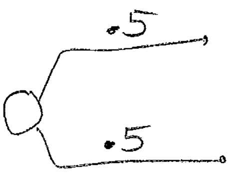


$U'(x) = b r e^{-rx}$, $U''(x) = -b r^2 e^{-rx}$

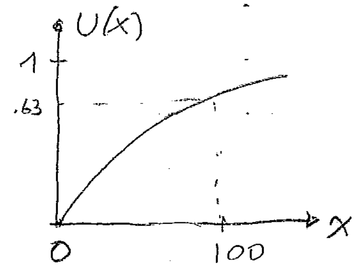
$$r(x) = -\frac{U''}{U'} = -\frac{-b r^2 e^{-rx}}{b r e^{-rx}} = r \quad : \text{Constant}$$

Constant risk aversion is not very common. But if an individual exhibits constant risk aversion, this implies that his RP depends on difference between outcomes, not absolute value of outcomes.

Ex. $U(x) = 1 - e^{-0.01x}$, $a = b = 1$, $r = 0.01$



	(1)	(2)	(3)
.5	30 $\langle .259\rangle$	50 $\langle .393\rangle$	70 $\langle .503\rangle$
.5	0 $\langle 0\rangle$	20 $\langle .181\rangle$	40 $\langle .329\rangle$



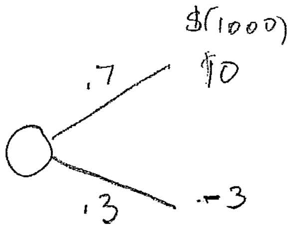
50/50 Gamble	EMV	CE	RP
0, 30	15	13.86	~1.16
20, 50	35	33.82	~1.16
40, 70	55	53.78	~1.16

Constant

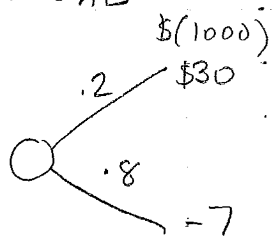
Ex. Separability with constant risk aversion

Assume constant risk aversion & Indep. events

A:



B:

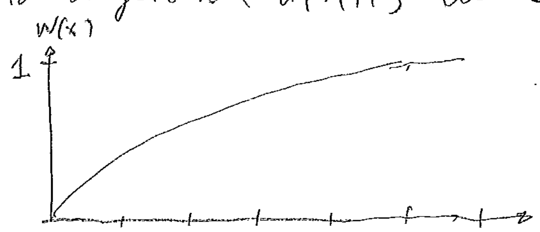


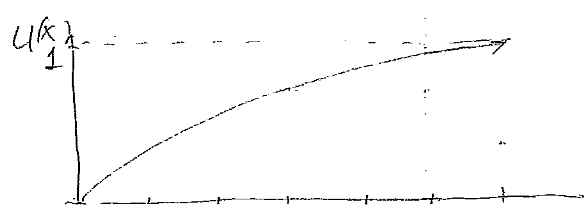
Which project (or both) should be taken?

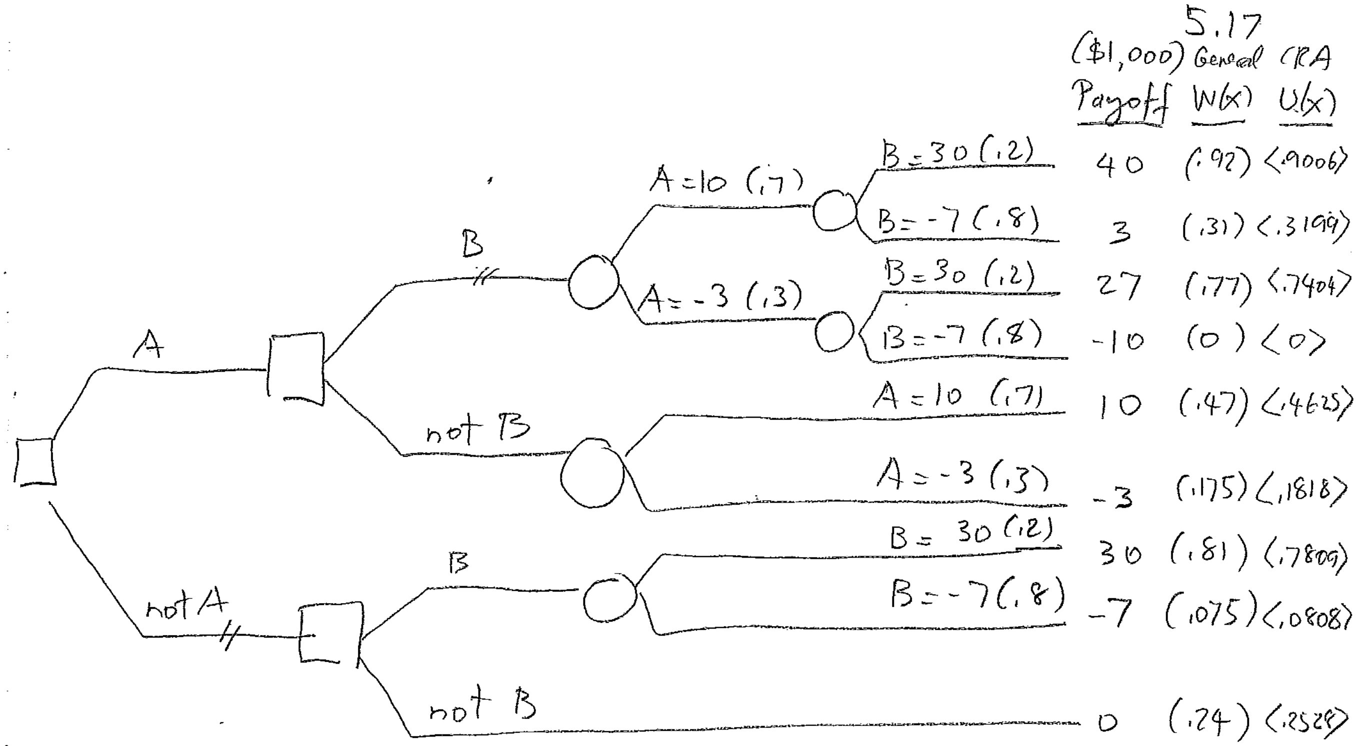
For the constant risk aversion ut. function use

$$U(x) = 1.48368 - 1.23087\, e^{-0.00001868x}$$

For a general utility curve use $W(x)$ in Fig. 16.18 (p407)







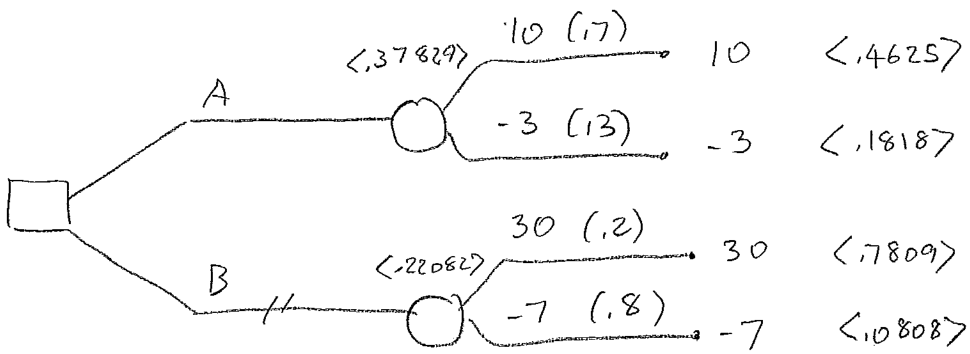
Certainty Equiv. with

	General W(x)	CRA U(x)
A and B	5,000	4,386
A	6,500	5,758
B	-1,000	-1,372
Neither A nor B	0	0

A+B ≠ A and B — 6,500-1,000 ≠ 5,000

A+B = A and B — 5,758-1,372 = 4,386

∴ With CRA utility we could have looked at the problem after separating A and B projects

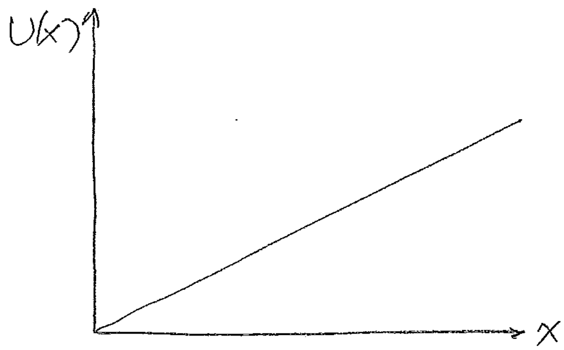


Show that A is better

Risk Neutrality

Corresponds to linear utility functions. We also have contant risk aversion, i.e. $r(x)=0 \ \forall x$.

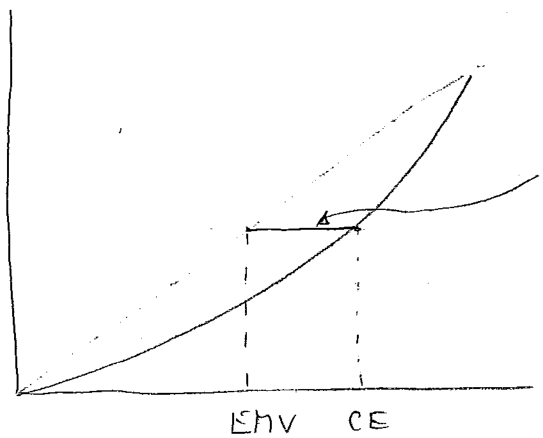
Since $CE = EMV$, we get $RP = 0$



$U(x) = ax+b.$

In general for small gambles which are repetitive, individuals are usually risk-neutral. Quality control policies, inventory control policies are usually made assuming risk-neutrality.

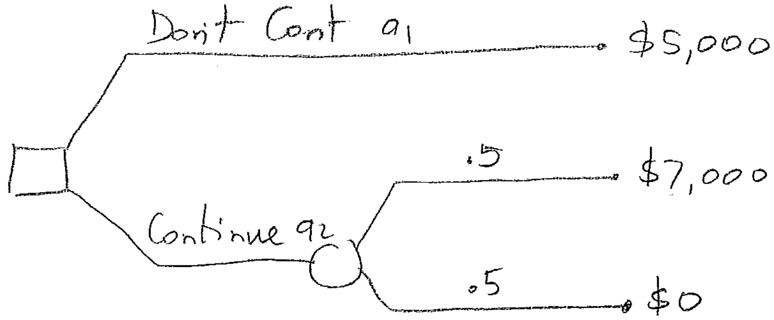
Risk Seeking



$RP = EMV - CE < 0$

Applicable to gamblers

Ex. Game Show

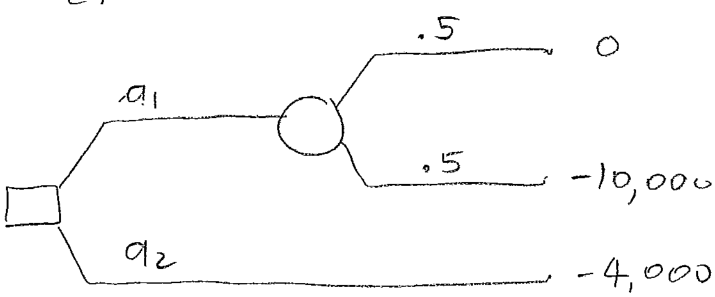


$EMV(a_1) = 5000$

$EMV(a_2) = 3{,}500$ ← Still chooses this despite the lower EMV

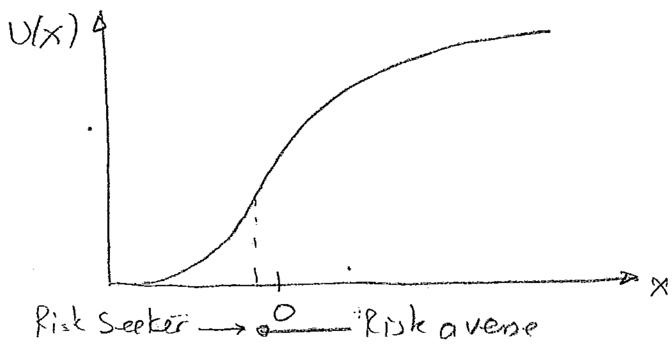
Risk-seeking is observed in settings where There's a Threshold. Once a certain amount of money is obtained, it leads to a new life

Individuals are also risk seekers for gambles with negative EMV.



$EMV(a_1) = -5{,}000$ ← Still chooses this

$EMV(a_2) = -4{,}000$



Friedman-Savage utility