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[ > restart: # Nucleolus-NonEmptyCore-ThreeCities.mws
> Nu:=proc(v12,v13,v23,v123)
  local y1, y2, y3;

  if v123>=3*v12 and v123>=3*v13 and v123>=3*v23 then
    y1:=v123/3;
    y2:=y1;
    y3:=y1;
    print("Case 1",v12,v13,v23,v123,[y1,y2,y3])

  elif v123>=v12+2*v23 and v123>=v12+2*v13 and v123<=3*v12 then
    y1:=(v123+v12)/4;
    y2:=y1;
    y3:=(v123-v12)/2;
    print("Case 2",v12,v13,v23,v123,[y1,y2,y3])

  elif v123>=v12+2*v23 and v123<=v12+2*v13 and v12>=v13 then
    y1:=(v12+v13)/2;
    y2:=(v123-v13)/2;
    y3:=(v123-v12)/2;
    print("Case 3",v12,v13,v23,v123,[y1,y2,y3])

  elif v123<=v12+2*v23 and v123>=v12+2*v13 and v23<=v12 then
    y1:=(v123-v23)/2;
    y2:=(v12+v23)/2;
    y3:=(v123-v12)/2;
    print("Case 4",v12,v13,v23,v123,[y1,y2,y3])

  elif v123<=v12+2*v23 and v123<=v12+2*v13 and v123+v12>=2*(v13+v23)
  then
    y1:=(v123+v12-2*(v23-v13))/4;
    y2:=(v123+v12+2*(v23-v13))/4;
    y3:=(v123-v12)/2;
    print("Case 5",v12,v13,v23,v123,[y1,y2,y3])

  elif v123>=v13+2*v23 and v123>=v13+2*v12 and v123<=3*v13 then
    y1:=(v123+v13)/4;
    y2:=(v123-v13)/2;
    y3:=y1;
    print("Case 6",v12,v13,v23,v123,[y1,y2,y3])

  elif v123>=v13+2*v23 and v123<=v13+2*v12 and v13>=v12 then
    y1:=(v12+v13)/2;

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y2:=(v123-v13)/2;
y3:=(v123-v12)/2;
print("Case 7",v12,v13,v23,v123,[y1,y2,y3])

elif v123<=v13+2*v23 and v123>=v13+2*v12 and v13>=v23 then
y1:=(v123-v23)/2;
y2:=(v123-v13)/2;
y3:=(v13+v23)/2;
print("Case 8",v12,v13,v23,v123,[y1,y2,y3])

elif v123<=v13+2*v23 and v123<=v13+2*v12 and v123+v13>=2*(v12+v23)
then
y1:=(v123+v13-2*(v23-v12))/4;
y2:=(v123-v13)/2;
y3:=(v123+v13+2*(v23-v12))/4;
print("Case 9",v12,v13,v23,v123,[y1,y2,y3])

elif v123>=v23+2*v13 and v123>=v23+2*v12 and v123<=3*v23 then
y1:=(v123-v23)/2;
y2:=(v123+v23)/4;
y3:=y2;
print("Case 10",v12,v13,v23,v123,[y1,y2,y3])

elif v123>=v23+2*v13 and v123<=v23+2*v12 and v23>=v12 then
y1:=(v123-v23)/2;
y2:=(v12+v23)/2;
y3:=(v123-v12)/2;
print("Case 11",v12,v13,v23,v123,[y1,y2,y3])

elif v123<=v23+2*v13 and v123>=v23+2*v12 and v23>=v13 then
y1:=(v123-v23)/2;
y2:=(v123-v13)/2;
y3:=(v13+v23)/2;
print("Case 12",v12,v13,v23,v123,[y1,y2,y3])

elif v123<=v23+2*v13 and v123<=v23+2*v12 and v123+v23>=2*(v13+v12)
then
y1:=(v123-v23)/2;
y2:=(v123+v23-2*(v13-v12))/4;
y3:=(v123+v23+2*(v13-v12))/4;
print("Case 13",v12,v13,v23,v123,[y1,y2,y3])

elif v123+v12<=2*(v13+v23) and v123+v13<=2*(v12+v23) and

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v123+v23<=2*(v13+v12) then
y1:=(v123+v12+v13-2*v23)/3;
y2:=(v123+v12+v23-2*v13)/3;
y3:=(v123+v13+v23-2*v12)/3;
print("Case 14",v12,v13,v23,v123,[y1,y2,y3])

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else "No good"
end if
end proc;

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N := proc(v12, v13, v23, v123)
local y1, y2, y3;
if 3*v12 ≤ v123 and 3*v13 ≤ v123 and 3*v23 ≤ v123 then
y1 := 1 / 3*v123; y2 := y1; y3 := y1; print("Case 1", v12, v13, v23, v123, [y1, y2, y3])
elif v12 + 2*v23 ≤ v123 and v12 + 2*v13 ≤ v123 and v123 ≤ 3*v12 then
y1 := 1 / 4*v123 + 1 / 4*v12;
y2 := y1;
y3 := 1 / 2*v123 - 1 / 2*v12;
print("Case 2", v12, v13, v23, v123, [y1, y2, y3])
elif v12 + 2*v23 ≤ v123 and v123 ≤ v12 + 2*v13 and v13 ≤ v12 then
y1 := 1 / 2*v12 + 1 / 2*v13;
y2 := 1 / 2*v123 - 1 / 2*v13;
y3 := 1 / 2*v123 - 1 / 2*v12;
print("Case 3", v12, v13, v23, v123, [y1, y2, y3])
elif v123 ≤ v12 + 2*v23 and v12 + 2*v13 ≤ v123 and v23 ≤ v12 then
y1 := 1 / 2*v123 - 1 / 2*v23;
y2 := 1 / 2*v12 + 1 / 2*v23;
y3 := 1 / 2*v123 - 1 / 2*v12;
print("Case 4", v12, v13, v23, v123, [y1, y2, y3])
elif v123 ≤ v12 + 2*v23 and v123 ≤ v12 + 2*v13 and 2*v13 + 2*v23 ≤ v123 + v12 then
y1 := 1 / 4*v123 + 1 / 4*v12 - 1 / 2*v23 + 1 / 2*v13;
y2 := 1 / 4*v123 + 1 / 4*v12 + 1 / 2*v23 - 1 / 2*v13;
y3 := 1 / 2*v123 - 1 / 2*v12;
print("Case 5", v12, v13, v23, v123, [y1, y2, y3])
elif v13 + 2*v23 ≤ v123 and v13 + 2*v12 ≤ v123 and v123 ≤ 3*v13 then
y1 := 1 / 4*v123 + 1 / 4*v13;
y2 := 1 / 2*v123 - 1 / 2*v13;
y3 := y1;
print("Case 6", v12, v13, v23, v123, [y1, y2, y3])
elif v13 + 2*v23 ≤ v123 and v123 ≤ v13 + 2*v12 and v12 ≤ v13 then
y1 := 1 / 2*v12 + 1 / 2*v13;

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    y2 := 1 / 2*v123 - 1 / 2*v13;
    y3 := 1 / 2*v123 - 1 / 2*v12;
    print("Case 7", v12, v13, v23, v123, [y1, y2, y3])
elif v123 ≤ v13 + 2*v23 and v13 + 2*v12 ≤ v123 and v23 ≤ v13 then
    y1 := 1 / 2*v123 - 1 / 2*v23;
    y2 := 1 / 2*v123 - 1 / 2*v13;
    y3 := 1 / 2*v13 + 1 / 2*v23;
    print("Case 8", v12, v13, v23, v123, [y1, y2, y3])
elif v123 ≤ v13 + 2*v23 and v123 ≤ v13 + 2*v12 and 2*v12 + 2*v23 ≤ v123 + v13 then
    y1 := 1 / 4*v123 + 1 / 4*v13 - 1 / 2*v23 + 1 / 2*v12;
    y2 := 1 / 2*v123 - 1 / 2*v13;
    y3 := 1 / 4*v123 + 1 / 4*v13 + 1 / 2*v23 - 1 / 2*v12;
    print("Case 9", v12, v13, v23, v123, [y1, y2, y3])
elif v23 + 2*v13 ≤ v123 and v23 + 2*v12 ≤ v123 and v123 ≤ 3*v23 then
    y1 := 1 / 2*v123 - 1 / 2*v23;
    y2 := 1 / 4*v123 + 1 / 4*v23;
    y3 := y2;
    print("Case 10", v12, v13, v23, v123, [y1, y2, y3])
elif v23 + 2*v13 ≤ v123 and v123 ≤ v23 + 2*v12 and v12 ≤ v23 then
    y1 := 1 / 2*v123 - 1 / 2*v23;
    y2 := 1 / 2*v12 + 1 / 2*v23;
    y3 := 1 / 2*v123 - 1 / 2*v12;
    print("Case 11", v12, v13, v23, v123, [y1, y2, y3])
elif v123 ≤ v23 + 2*v13 and v23 + 2*v12 ≤ v123 and v13 ≤ v23 then
    y1 := 1 / 2*v123 - 1 / 2*v23;
    y2 := 1 / 2*v123 - 1 / 2*v13;
    y3 := 1 / 2*v13 + 1 / 2*v23;
    print("Case 12", v12, v13, v23, v123, [y1, y2, y3])
elif v123 ≤ v23 + 2*v13 and v123 ≤ v23 + 2*v12 and 2*v12 + 2*v13 ≤ v123 + v23 then
    y1 := 1 / 2*v123 - 1 / 2*v23;
    y2 := 1 / 4*v123 + 1 / 4*v23 - 1 / 2*v13 + 1 / 2*v12;
    y3 := 1 / 4*v123 + 1 / 4*v23 + 1 / 2*v13 - 1 / 2*v12;
    print("Case 13", v12, v13, v23, v123, [y1, y2, y3])
elif v123 + v12 ≤ 2*v13 + 2*v23 and v123 + v13 ≤ 2*v12 + 2*v23 and
v123 + v23 ≤ 2*v12 + 2*v13 then
    y1 := 1 / 3*v123 + 1 / 3*v12 + 1 / 3*v13 - 2 / 3*v23;
    y2 := 1 / 3*v123 + 1 / 3*v12 + 1 / 3*v23 - 2 / 3*v13;
    y3 := 1 / 3*v123 + 1 / 3*v13 + 1 / 3*v23 - 2 / 3*v12;
    print("Case 14", v12, v13, v23, v123, [y1, y2, y3])
else "No good"
end if

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end proc
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[ > evalf(Nu(90,100,120,220));  
      "Case 14", 90, 100, 120, 220,  $\left[\frac{170}{3}, \frac{230}{3}, \frac{260}{3}\right]$   
[ > evalf([170/3, 230/3, 260/3]);  
      [56.66666667, 76.66666667, 86.66666667]  
[ >  
[ >
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