Game-Theoretic Models in Supply Chain Management

2012 PhD Summer Academy

Zaragoza Logistics Center, SPAIN

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Final Exam (Take home)

- **Due date: July 2, 2012 (Monday), 1:00 p.m. in Room 317**
- **Each question is assigned 25 marks.**

1. Vickrey auction: For this problem, we define the following. Let S_i denote player i's strategy set $(i = 1, \dots, n)$, and $V_i(s_i, s_{-i})$ be the payoff of player i when his strategy is s_i and other players use $s_{-i} = (s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_n)$. A strategy s'' **weakly dominates** a strategy s' if and only if,

$$\begin{aligned} V_i(s_i'', s_{-i}) &\geq V_i(s_i', s_{-i}) \text{ for all } s_{-i} \in S_{-i}, \text{ and} \\ V_i(s_i'', s_{-i}) &> V_i(s_i', s_{-i}) \text{ for some } s_{-i} \in S_{-i}. \end{aligned}$$

Now consider the **first-price auction** with two bidders. Player P1 values the item to be auctioned as \$4 and P2 values it as \$3. (We assume complete information in this problem.) The bidder with the higher bid wins the item and pays the higher bid. When there is a tie, assume that the winner is determined using the result of tossing a fair coin. The payoffs are calculated as the difference between the valuation of a player and the actual payment. For example, if P1 pays 3 and P2 pays 1, P1 wins and pays 3 resulting in a payoff of $4 - 3 = 1$ for P1 and 0 for P2 since she does not win.

(a) For the first-price auction, show that the payoff matrix for the two players is given as,

		P2				
		1	2	3	4	5
	1	$\frac{3}{2}, 1$	$0, 1$	$0, 0$	$0, -1$	$0, -2$
	2	$2, 0$	$1, \frac{1}{2}$	$0, 0$	$0, -1$	$0, -2$
P1	3	$1, 0$	$1, 0$	$\frac{1}{2}, 0$	$0, -1$	$0, -2$
	4	$0, 0$	$0, 0$	$0, 0$	$0, -\frac{1}{2}$	$0, 2$
	5	$-1, 0$	$-1, 0$	$-1, 0$	$-1, 0$	$-\frac{1}{2}, -1$

Start by eliminating all weakly-dominated strategies. Then find the Nash equilibrium for this game. Is it truth- revealing, i.e., will the bidders bid their true valuations?

(b) Now consider the **second-price** Vickrey auction where the higher bidder wins but pays the lower bid. For example, if P1 bids 3 and P2 bids 1, then P1 wins and pays 1, hence his payoff is 3 (difference between 4 and 1) and P2's payoff is 0. (Ties are broken as in the first-price auction.) Calculate the payoffs and eliminate all weakly-dominated strategies. Show that in this case bidding one's true valuation (i.e., 4 and 3) is the Nash equilibrium.

2. Consider the Battle-of-the-Sexes problem with one-sided incomplete information discussed in class. As you will recall, in this problem the Nature selects the women's type ("yes" or "no") with equal probabilities, i.e., $p(y) = p(n) = \frac{1}{2}$. In this problem, the payoff matrices were,

$$y: \begin{pmatrix} 2,1 & 0,0 \\ 0,0 & 1,2 \end{pmatrix}, \quad n: \begin{pmatrix} 2,0 & 0,2 \\ 0,1 & 1,0 \end{pmatrix},$$

and the Bayesian Nash equilibrium (BNE) was $(S, \overset{y}{S}\overset{n}{B})$ where S is "soccer" and B is "ballet."

Now consider the same problem but assume that the Nature's probabilities are $p(y) = \rho$ and $p(n) = 1 - \rho$. Show that as long as $\frac{1}{3} < \rho < \frac{2}{3}$, the above BNE does not change. (Hint: Write the man's payoffs in the 2×4 strategic form in terms of the parameter ρ.)

3. Refer to Section 3.2.1 of the paper by Wu and Parlar [3]. Now set the parameters as $[a, b \mid s_1, s_2 \mid c_2] = [0.9, 0.9 \mid 15, 15 \mid 8]$, but now since the unit purchase cost of the first newsvendor could be low or high, we let $[c_{1L}, c_{1H}] = [6, 10]$. Demand densities are exponential, i.e., $f(x) = \lambda e^{-\lambda x}$ and $h(y) = \mu e^{-\mu y}$ with respective parameters $(\lambda, \mu) = \left(\frac{1}{30}, \frac{1}{30}\right)$. Find the Bayesian Nash equilibrium for this problem.

4. Consider Example 1 of Section 5 in Leng and Parlar [1]. In this example, we denote the Manufacturer, Distributor and Retailer as 1, 2 and 3, respectively, where the characteristic function values are $v(12) = 12.31$, $v(13) = 19.67$, $v(23) = 21.62$, and $v(123) = 42.68$.

   (a) Briefly review the contribution of the Leng and Parlar [1] paper.

   (b) Is the core empty or non-empty for this cooperative game? Explain.

   (c) Find the Shapley value.

   (d) Find the nucleolus using Leng and Parlar's analytic results in [2].

References

[1] M. Leng and M. Parlar. Allocation of cost savings in a three-level supply chain with demand information sharing: A cooperative-game approach. *Operations Research*, 57(1):200–213, January–February 2009.

[2] M. Leng and M. Parlar. Analytic solution for the nucleolus of a three-player cooperative game. *Naval Research Logistics*, 57:667–672, 2010.

[3] H. Wu and M. Parlar. Games with incomplete information: A simplified exposition with inventory management applications. *International Journal of Production Economics*, 133:562–577, 2011.