2012-06-27

Wednesday, June 27, 2012

10:16 AM

Final due July 2 (Monday) 1:00 pm
in office

$(S, \overset{y}{S}\overset{n}{B})$

let's formalize the BNE

$\sigma_i(t_i)$: strategy profile for P_i and $t_i \in T_i$

$$\hat{J}_1(\sigma_1(t_1), \sigma_2(t_2), t_1) = \sum_{t_2 \in T_2} J_1(\sigma_1(t_1), \sigma_2(t_2); t_1, t_2)\, p_1(t_2 | t_1)$$

for all $t_1 \in T_1$

$$\hat{J}_2(\sigma_1(t_1), \sigma_2(t_2); t_2) = \sum_{t_1 \in T_1} J_2(\sigma_1(t_1), \sigma_2(t_2); t_1, t_2)\, p_2(t_1 | t_2)$$

for all $t_2 \in T_2$

A strategy profile $(\sigma_1^*(t_1), \sigma_2^*(t_2))$ is Bayesian Nash equilib. (BNE) if

$$\hat{J}_1(\sigma_1^*(t_1), \sigma_2^*(t_2), t_1) \geq \hat{J}_1(\sigma_1'(t_1), \sigma_2^*(t_2), t_1)$$

$$\hat{J}_2(\sigma_1^*(t_1), \sigma_2^*(t_2), t_2) \geq \hat{J}_2(\sigma_1^*(t_1), \sigma_2'(t_2), t_2)$$

for all $t_i \in T_i$ and alternat. strategy $\sigma_i'(t_i)$ for P_i

Back to BofS

man:

$$\hat{J}_1(S, SS; 1) = J_1(S, SS; 1, y)\, p(y \mid 1) + J_1(S, SS; 1, n)\, p(n \mid 1)$$
$$= 2 \cdot \frac{1}{2} + 2 \cdot \frac{1}{2} = 2$$

woman

$$\hat{J}_2(S, SS; y) = J_2(S, SS; 1, y)\, p(1 \mid y)$$
$$= 1 \cdot 1 = 1$$

$$\hat{J}_2(S, SS; n) = J_2(S, SS; 1, n)\, p(1 \mid n)$$
$$= 0 \cdot 1 = 0$$

$\vdots$

BNE: $(S, \underset{y}{S}\underset{n}{B})$

Ex. Cournot competition under asymmetry

As before $P(Q) = a - Q$, $Q = q_1 + q_2$

Firm 1

$$C_1(q_1) = c q_1$$

Firm 2

$$C_2(q_2) = \begin{cases} c_H q_2, & \theta \\ c_L q_2, & 1-\theta \end{cases} \qquad c_L < c_H$$

$$q_{2H} = q_2(c_H)$$
$$q_{2L} = q_2(c_L)$$

Firm 1 (q_1)

$$\hat{J}_1(\sigma_1(t_1), \sigma_2(t_2); t_1) = \sum_{t_2 \in T_2} J_1(\sigma_1(t_1), \sigma_2(t_2); t_1, t_2)\, p_1(t_2 | t_1)$$

$$= q_1\left[(a - (q_1 + q_{2H})) - c\right] \overset{\theta}{\overset{\|}{p_1(c_H | c)}}$$

$$+ q_1\left[(a - (q_1 + q_{2L})) - c\right] p_1(c_L | c)$$

$$= \boxed{q_1\left[(a - (q_1 + q_{2H})) - c\right]\theta + q_1\left[(a - (q_1 + q_{2L})) - c\right](1-\theta)}$$

Firm 2 (c_H)

$$\hat{J}_{2H}(\sigma_1(t_1), \sigma_2(t_2); \overset{c_H}{\overset{\|}{t_2}}) = \sum_{t_1 \in T_1} J_1(\sigma_1(t_1), \sigma_2(t_2); t_1, t_2)\, \underbrace{p_2(\overset{c}{\overset{\|}{t_1}} | \overset{c_H}{\overset{\|}{t_2}})}_{1}$$

$$= \boxed{\left[(a-(q_1+q_{2H})-c_H\right]\cdot q_{2H}} \cdot 1$$

Firm 2 (c_L)

$$\hat{J}_{2L}(\sigma_1(t_1),\sigma_2(t_2),\underset{c_L}{t_2}) = \sum_{t_1\in T_1} \left(\qquad\qquad \right) \underbrace{p_2(\overset{c}{t_1}|\overset{c_L}{t_2})}_{1}$$

$$= \boxed{\left[(a-(q_1+q_{2L})-c_L\right]\cdot q_{2L}} \cdot 1$$

Solve $\quad \dfrac{\partial \hat{J}_1}{\partial q_1}=0, \quad \dfrac{\partial \hat{J}_{2H}}{\partial q_{2H}}=0, \quad \dfrac{\partial \hat{J}_{2L}}{\partial q_{2L}}=0$

$$\Rightarrow \left.\begin{aligned} &-2q_1+\theta\left[a-q_H-c\right]+(1-\theta)\left[a-q_{2L}-c\right]=0 \\ &-2q_{2H}+a-q_1-c_H=0 \\ &-2q_{2L}+a-q_1-c_L=0 \end{aligned}\right\}$$

$$\Rightarrow \quad q_1^* = \frac{1}{3}\left\{a-2c+\left[\overbrace{\theta c_H+(1-\theta)c_L}^{\text{convex comb}}\right]\right\}$$

$$q_{2H}^* = \frac{1}{3}(a-2c_H+c)+\frac{1-\theta}{6}(c_H-c_L)$$

$$q_{2L}^* = \frac{1}{3}(a - 2c_L + c) - \frac{\theta}{6}(c_H - c_L)$$

Ex. Two news vendor

P1		P2
c_{1L}	c_{1H}	c_2
q_{1L}	q_{1H}	q_2

$$\hat{J}_{1L}(\sigma_1(t_1), \sigma_2(t_2)) = J_{1L}(q_{1L}, q_2)$$

$$\hat{J}_{1H}(\qquad) = J_{1H}(q_{1H}, q_2)$$

$$\hat{J}_2(\qquad) = \theta_1 J_2(q_{1L}, q_2) + (1-\theta) J_2(q_{1H}, q_2)$$

Same numerical values as before

$$[a, b \mid s_1, s_2 \mid c_2] = (.5, .9 \mid 15, 9 \mid 5)$$

$$\underset{(8)}{c_i} : \Rightarrow \quad (c_{1L}, c_{1H}) = (6, 10)$$

<u>Soln</u>

	q_{1L}	q_{1H}	q_2	$\hat{J}_{1L}$	$\hat{J}_{1H}$	$\hat{J}_2$
Bayesian Nash	35	17	20	44	41	36

Baysian Nash 35 17 20 144 41 36

Nash $q_1 = 25$ 19 $J_1 = 83$ 35

III. Mechanism Design (Theory of Incentives)

① less-informed (Principal) P

② more- " (Agent) A

Adverse Selection: · Hidden knowledge of A

- Major hiway

	State	Contractor
	P	A

θ: cost to produce + deliver (type)

q: # lanes to be built (Laffont + Martimort)

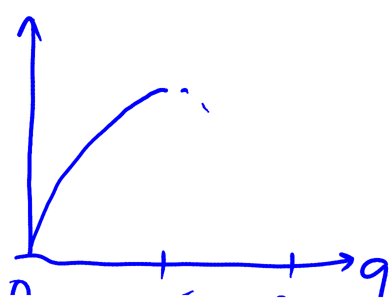
$S(q)$: value $\quad S(q) = 15q - \frac{1}{2}q^2$

$$= \left(15 - \frac{1}{2}q\right)q$$

$S'(q) > 0$

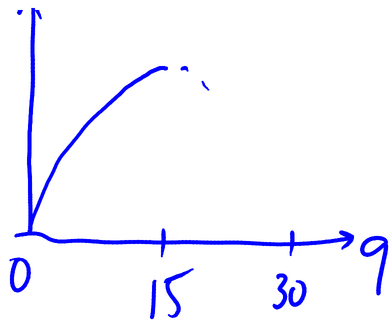
$S''(q) < 0$

$S(0) = 0$



$S''(q) < 0$

$S(0) = 0$



(i) Basic model

θ: Cost of constr. / lane (including profit)

$\Theta = \{\theta_1, \theta_2\}$
3   5 mill.

$\Delta\theta = \theta_2 - \theta_1 = 2$ spread of uncertainty

So,

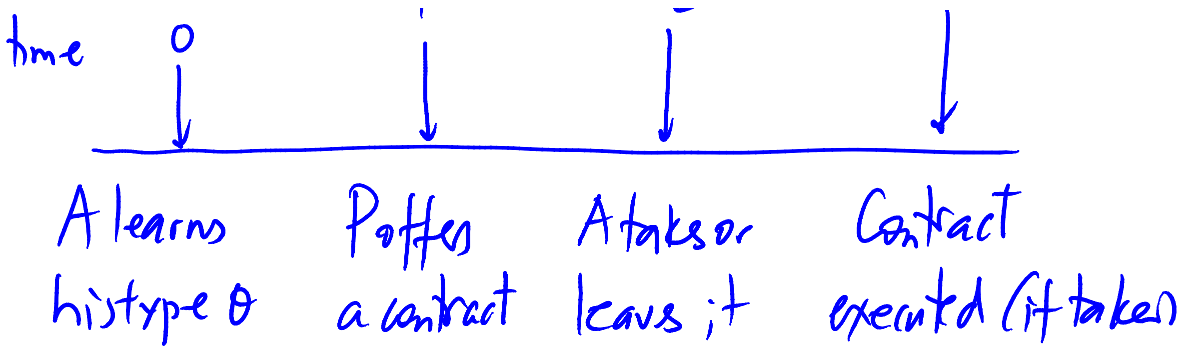
$$c(q, \theta) = \begin{cases} \theta_1 q & \text{with } \rho \\ \theta_2 q & \text{"} \ 1-\rho \end{cases}$$

$\mathcal{A}$: set of allocations

$$\mathcal{A} = \{(q, t) : q \in \mathbb{R}^+, t \in \mathbb{R}\}$$

t: transfer to agent

time 0 | 1 | 2 | 3



(ii) Complete info + "first-best" production

Social value $\quad W = S(q) - C(q,\theta)$

$$W' = S'(q) - C'(q,\theta)$$
$$= S'(q) - \theta = 0 \qquad \boxed{S'(q) = \theta}$$

$\theta_1 = 3: \qquad S'(q_1^*) = \theta_1 : \quad 15 - q = 3 \Rightarrow q_1^* = 12$

$\theta_2 = 5: \qquad S'(q_1^*) = \theta_2 : \quad 15 - q = 5 \Rightarrow q_2^* = 10$

How much to pay? $\quad t = (t_1, t_2)$

Participation constraint $\qquad t_1 - \theta_1 q_1 \geq 0$

$$t_2 - \theta_2 q_2 \geq 0$$

So, for each type $i=1,2$

$$\left.\begin{aligned}&\max_{q,t}\ S(q)-t\\ &\quad \text{s.t. } t-\theta q \geq 0\end{aligned}\right\}$$

$$\mathcal{L}=S(q)-t-\lambda(\theta q-t)$$

$$\mathcal{L}_q=\boxed{S'(q)-\lambda\theta=0}$$

$$\mathcal{L}_t=-1+\lambda=0 \Rightarrow \lambda=1$$

$$\lambda\geq 0,\quad \theta q-t\leq 0$$

$$\lambda\underbrace{(\theta q-t)}_{0}=0 \Rightarrow \boxed{t=\theta q}$$

Optimal Prod. $S'(q)=\theta$: $15-q=\theta$

Optimal payment $t_i=\theta_i q_i \begin{cases}36, & i=1\\ 50, & i=2\end{cases}$

Contract. $$\begin{array}{ll} & \quad\ q \quad\ t \\ A^* = & (12, 36) \\ B^* = & (10, 50)\end{array}$$

Information rent $U_1=t_1-\theta_1 q_1$

$$U_2 = t_2 - \theta_2 q_2$$

$$t \quad - \quad \theta q$$

θ_1 agent : at B^*, $U_1 = 50 - 3\cdot 10 = 20$ ✓

at A^* $U_1 = 36 - 3\cdot 12 = 0$

θ_2 agent : at B^*: $U_2 = 50 - 5\cdot 10 = 0$ ✓

at A^*: $U_2 = 36 - 5\cdot 12 = -24$

Conclusion: The menu (A^*, B^*) is a failure

θ_1 mimics θ_2 & selects B^*

$\{(t_1^*, q_1^*), (t_2^*, q_2^*)]$ is <u>not</u> incentive compatible

(iii) Incentive feasible menu

<u>Def</u> Incentive compatibility constraints

$$t_1 - \theta_1 q_1 \geq t_2 - \theta_1 q_2 \qquad (IC_1)$$

$$t_2 - \theta_2 q_2 \geq t_1 - \theta_2 q_1 \qquad (IC_2)$$

Incent. feasib.

Participation Const

$$t_1 - \theta_1 q_1 \geq 0 \qquad (PC_1)$$

$$t_2 - \theta_2 q_2 \geq 0 \qquad (PC_2)$$

Remark. The complete info sol

$\theta_1 = 3 \qquad (t_1, q_1) = (36, 12)$

$\theta_2 = 5 \qquad (t_2, q_2) = (50, 10)$

don't satisfy above

	LHS		RHS
IC_1:	$36 - 3 \cdot 12 = 0$	$\not\geq$	$50 - 3 \cdot 10 = 20$
IC_2:	$50 - 5 \cdot 10 = 0$	$>$	$36 - 5 \cdot 12 = -24$

(iv) P's opt. problem ("second-best")

ipsilon $\upsilon = Pr(\theta_1)$, $1 - \upsilon = Pr(\theta_2)$

$$(P) \quad \max_{\{(t_1, q_1), (t_2, q_2)\}} \upsilon [S(q_1) - t_1] + (1 - \upsilon)[S(q_2) - t_2]$$

$$\text{s.t. } IC_1, IC_2, PC_1, PC_2$$

[illegible]

Rewrite, using $U_1 = t_1 - \theta_1 q_1$

$U_2 = t_2 - \theta_2 q_2$

$$\max_{\{(U_1,q_1),(U_2,q_2)\}} \quad \overbrace{\nu\left[S(q_1)-\theta_1 q_1\right]+(1-\nu)\left[S(q_2)-\theta_2 q_2\right]}^{\text{exp. social value}} - \underbrace{\left[\nu U_1+(1-\nu)U_2\right]}_{\text{exp. info. rent}}$$

s.t.

$U_1 \geq U_2 + q_2 \Delta\theta \quad (=) \quad IC_1$

$U_2 \geq U_1 - q_1 \Delta\theta \quad (>) \quad IC_2$

$U_1 \geq 0 \quad (>) \quad PC_1$

$U_2 \geq 0 \quad (=) \quad PC_2$

Sol'n $\quad \nu = \frac{2}{3}, \quad 1-\nu = \frac{1}{3}$

$\hat{U}_1 = 12 \qquad \begin{bmatrix} \hat{q}_1 = 12 & \hat{t}_1 = 48 \\ \hat{q}_2 = 6 & \hat{t}_2 = 30 \end{bmatrix} \qquad P = 54$

$\hat{U}_2 = 0$

IC_1 $\quad 12 = 12$ ✓

IC_2 $\quad 0 > -12$ ✓

PC_1 $\quad 12 > 0$ ✓

PC_2 $\quad 0 = 0$ ✓

Reported type

	excess $t-\theta q$	θ_1	θ_2
True	$\theta_1 = 3$	$48-3\cdot 12=12$	$30-3\cdot 6=12$
	$\theta_2 = 5$	$48-5\cdot 12=-12$	$30-5\cdot 6=0$