2012-06-25

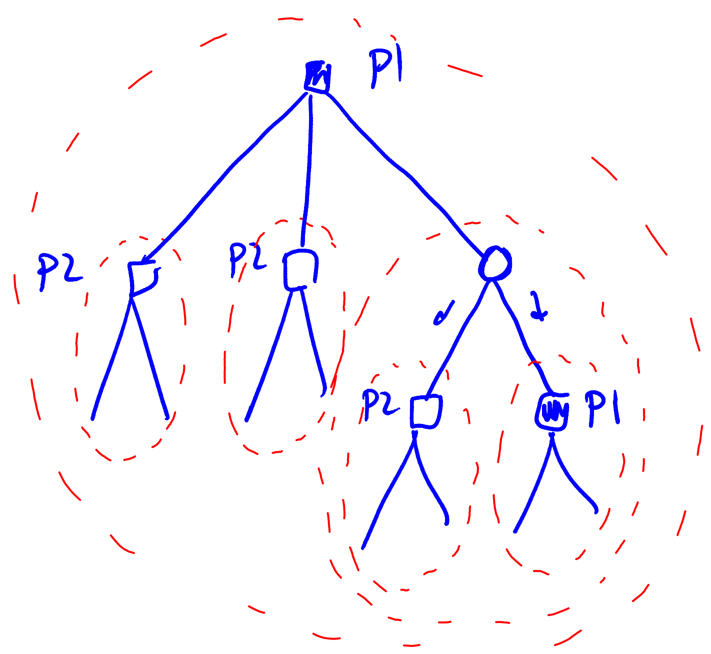
Monday, June 25, 2012

9:50 AM

II. "Dynamic" games with complete info
(Subgame perfect equilibrium - SPE)

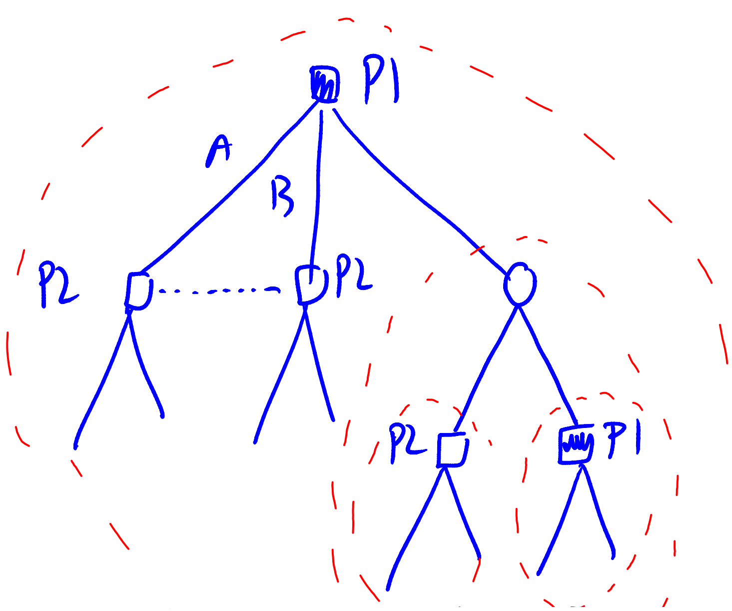
Def A subgame is any part of a game tree starting at a single decision (or chance) node which contains all successor nodes

Ex

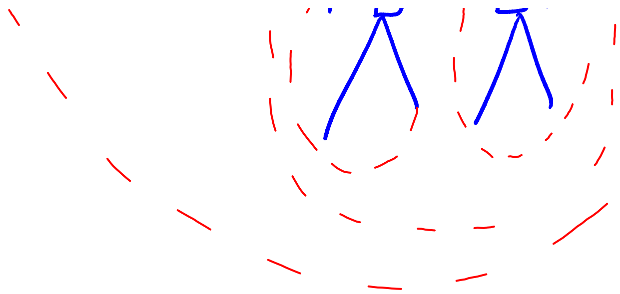


6 subgames

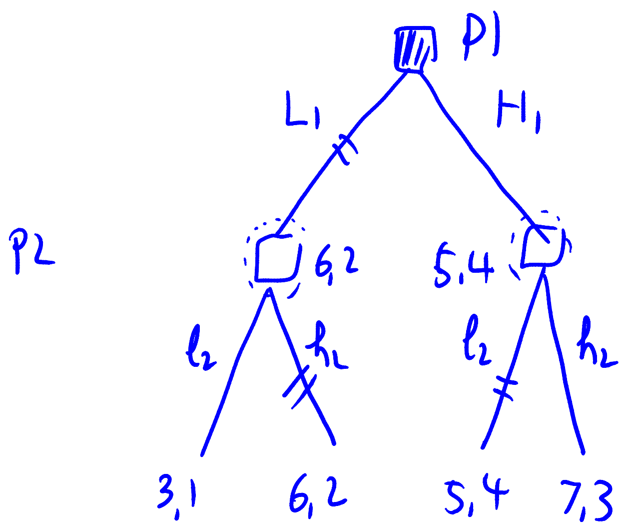
Ex



4 subgames



Ex. Discrete strategies (Hu + Parlar '11)



Strategies: A complete plan to play the game

P1: $\left.\begin{array}{l}1 \text{ inf. set} \\ 2 \text{ act}\end{array}\right\} \; 1\times 2 = 2$

P2. $\left.\begin{array}{l}2 \text{ inf. sets} \\ 2 \text{ act}\end{array}\right\} \; 2\times 2 = 4$

	L_1	H_1	
P1:	L_1	H_1	
P2:	l_2	l_2	: always low
	l_2	h_2	

$h_2 \quad l_2$

$h_2 \quad h_2$: always h_2

	$L_1 H_1$ $l_2 l_2$	$L_1 H_1$ $l_2 h_2$	$L_1 H_1$ $h_2 l_2$	$L_1 H_1$ $h_2 h_2$
L_1	$(3,1)$	$(3,1)$	$(\bar{6},\bar{2})$	$(6,\bar{2})$
H_1	$(\bar{5},\bar{4})$	$(\bar{7},3)$	$(5,\bar{4})$	$(\bar{7},3)$

Best response
$$R_2(q_1) = \begin{cases} h_2 & q_1 = L_1 \\ l_2 & q_1 = H_1 \end{cases}$$

Subgame perfect eq. (SPE): Combination of strategies for both players that result in a Nash eq. in every subgame

SPE: $(L_1, \underbrace{h_2 l_2}^{L_1 \; H_1})$

Backward induction equilibrium

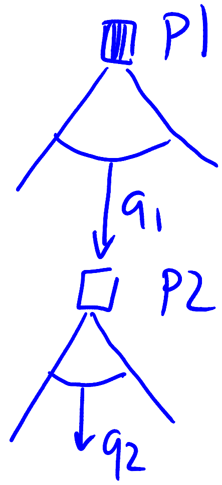
" " outcome: (L_1, h_2)

Stackelberg

Ex. Cont. strategies (Newsboy) w/o Parlar '11

$J_1(q_1,q_2)$ $\quad$ $J_2(q_1,q_2)$

leader $\quad$ follower



Best response for P2

$$\max_{q_2} J_2(q_1,q_2) \quad \text{for each } q_1$$

i.e., solve

$$\frac{\partial J_2}{\partial q_2} = \boxed{I_2(q_1,q_2) = 0}$$

Or,

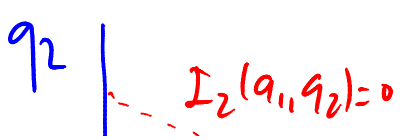
$$R_2(q_1) = \arg\max_{q_2 \geq 0} J_2(q_1,q_2)$$

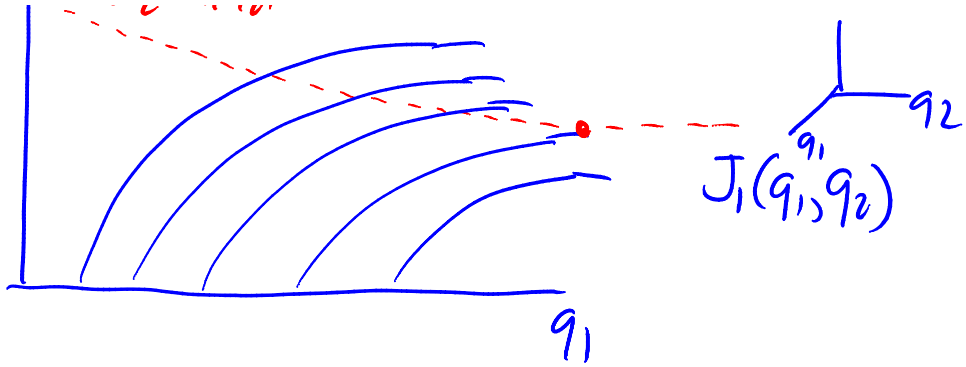
$$= \{ q_2 : I_2(q_1,q_2) = 0 \}$$

So, P1 has a constrained opt pb

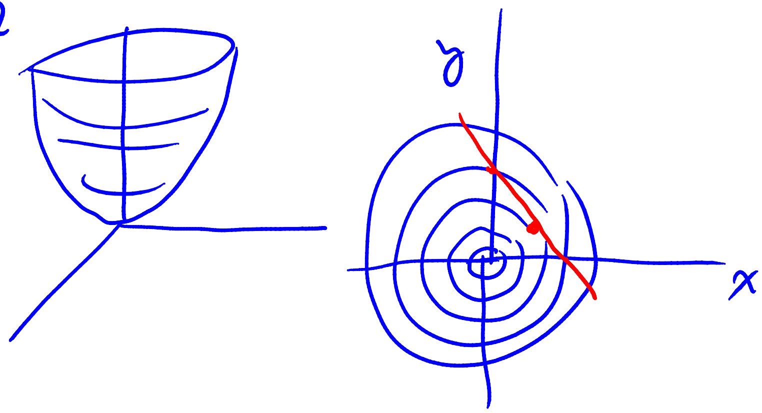
$$\max_{q_1 \geq 0} J_1(q_1,q_2)$$

$$\text{s.t. } I_2(q_1,q_2) = 0$$





$J(x,y) = x^2 + y^2$
s.t. $x + y = 1$



Same data $(q_1^s, q_2^s) = (28.38, 18.60)$

Stackelberg $(J_1^s, J_2^s) = (84.35, 33.94)$
(SPE)

Compare Nash $(q_1^N, q_2^N) = (25, 19)$

$(J_1^N, J_2^N) = (83, 35)$

Ex. Cournot duopoly

As before, $\pi_i = q_i [P(Q) - c]$, $P(Q) = a - Q$

$Q = q_1 + q_2$

$R_2(q_1)$: $\max_{q_2 \geq 0} \Pi_2(q_1, q_2) = \max_{q_2 \geq 0} q_2 [a - (q_1 + q_2) - c]$

$\Rightarrow \quad R_2(q_1) = \frac{1}{2}(a - q_1 - c) \qquad (q_1 < a - c)$

P1 anticipates

$$\max_{q_1 \geq 0} \Pi_1(q_1, R_2(q_1)) = \max_{q_1 \geq 0} q_1 [a - q_1 - R_2(q_1) - c]$$

$$= \max \frac{1}{2} q_1 (a - q_1 - c)$$

$\Rightarrow \quad q_1^* = \frac{1}{2}(a-c), \quad R_2(q_1^*) = \frac{1}{4}(a-c)$

Comparison

Cournot (NE)	Stackelberg (SPE)
$q_1^N = q_2^N = \frac{1}{3}(a-c)$	$q_1^S = \frac{1}{2}(a-c) > q_1^N$
	$q_2^S = \frac{1}{4}(a-c) < q_2^N$
$\pi_1^N = \pi_2^N = \frac{1}{9}(a-c)^2$	$\pi_1^S = \frac{1}{8}(a-c)^2 > \pi_1^N$
	$\pi_2^S = \frac{1}{16}(a-c)^2 < \pi_2^N$

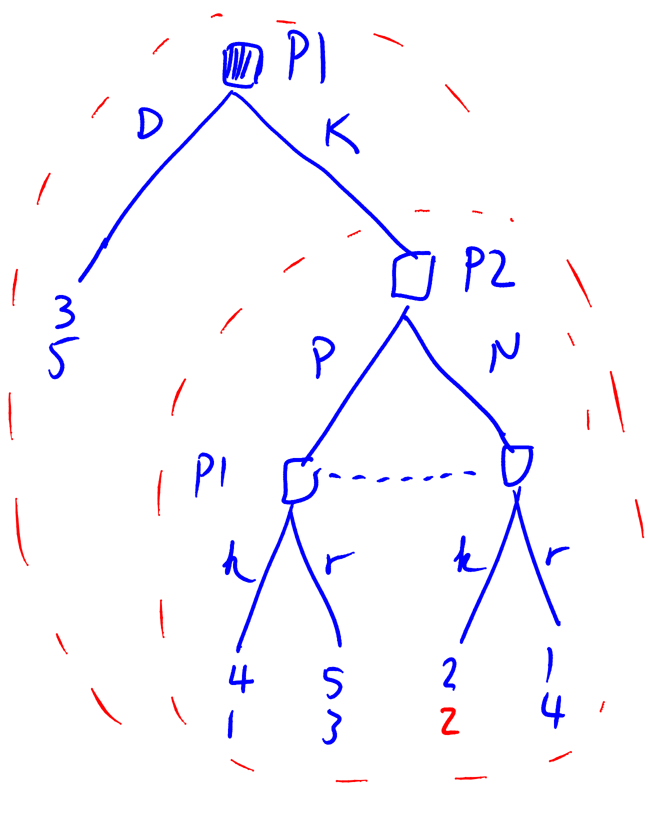
$$P^N = \frac{1}{3}a + \frac{2}{3}c \quad > \quad P^S = \frac{1}{4}a + \frac{3}{4}c$$

$$Q^N = \frac{2}{3}(a-c) \quad < \quad Q^S = \frac{3}{4}(a-c)$$

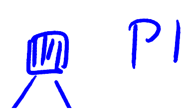
Pareto: $$\max_{q_1, q_2} \ \Pi_1(q_1,q_2) + \Pi_2(q_1,q_2)$$

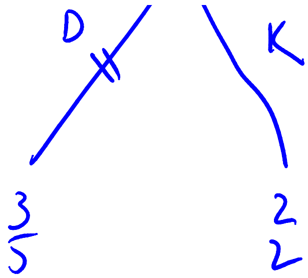
$$q_1^P = q_2^P = \frac{1}{4}(a-c) \ , \quad \Pi_1^P = \Pi_2^P = \frac{1}{8}(a-c)^2 > \Pi_i^N, \ i=1,2$$

Ex. Imperfect Info



	P	N
k	4,1	2,2
r	5,3	1,4



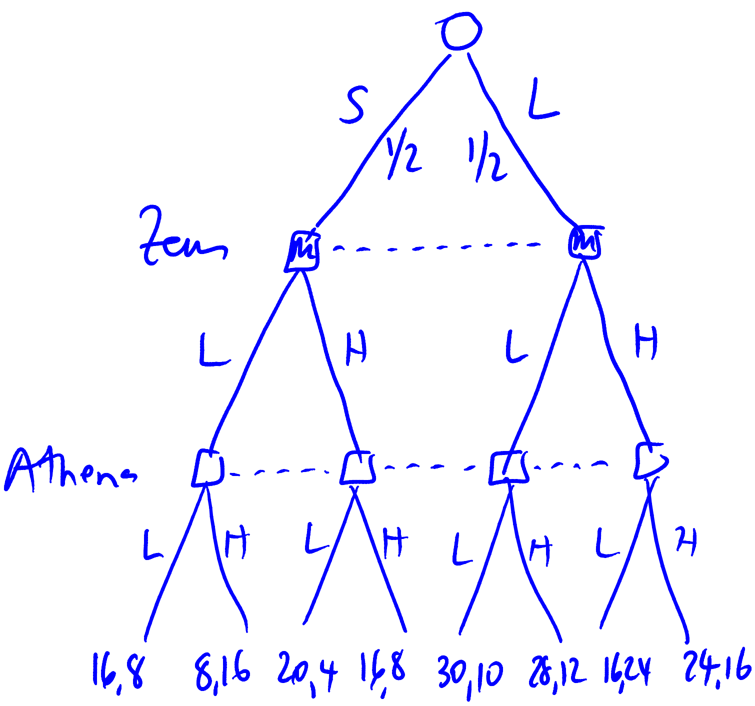


III. Static games of incomplete info (Bayesian Nash eq.)

(i) Extensive form & strategic form

Ex. Competition (constant sum game)

a) Simult. decision (still complete info)



$$
\begin{array}{ccc}
 & & \text{Ath} \\
 & & \begin{array}{cc} L \quad\quad & \quad\quad H \end{array} \\
\text{Zeus} & \begin{array}{c} L \\ \\ H \end{array} & \begin{pmatrix} \overline{23},9 & 18,\overline{14} \\ \\ 18,\overline{14} & \overline{20},12 \end{pmatrix}
\end{array}
$$

$LL: \frac{1}{2}(16,8)+\frac{1}{2}(30,10)$

$=(23,9)$

No pure

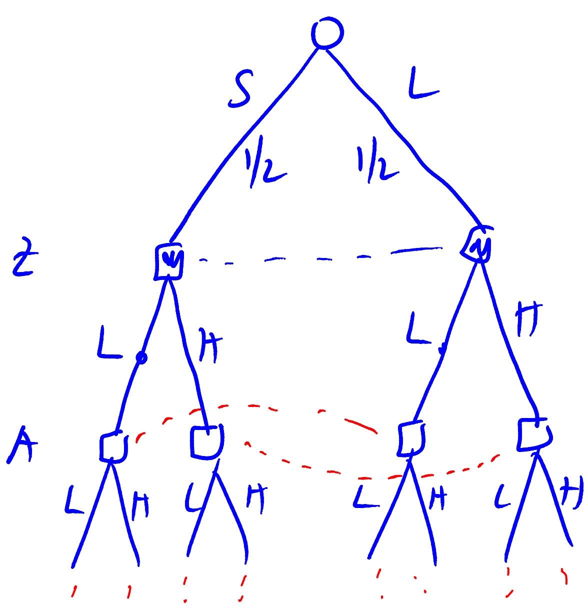
mixed $\left(\frac{2}{7},\frac{5}{7}\right)$

$Z=19\frac{3}{7}$

$A=12\frac{4}{7}$

b) Zeus first

Zeus	L	H
Ath	L	L
	L	H
	H	L
	H	H



		Zen → LH	LH	LH	LH
		LL	LH	HL	HH
Z	L	$\overline{23},9$	$\overline{23},9$	$\overline{18},\overline{14}$	$18,\overline{14}$
	H	$18,\overline{14}$	$20,12$	$\overline{18},\overline{14}$	$\overline{20},12$

Z = 18 , A = 14

(ii) Player types

Set of players $N = \{1, 2, \dots, n\}$

T_i : Set of types for player i

$$T = T_1 \times T_2 \times \cdots \times T_n = \{(t_1, \dots, t_n) : t_i \in T_i, \ i = 1, \dots, n\}$$

A game of incomplete info → a separate game for each poss. combination $t = (t_1, \dots, t_n) \in T$

Pi knows his type t_i, and

$$p(t_1, \dots, t_{i-1}, t_{i+1}, \dots, t_n \mid t_i) = p(t_{-i} \mid t_i), \quad \forall t_{-i} \in T_{-i}$$

Start with common joint pdf $p(\underbrace{t_1, \dots, t_n}_{t})$

$$p(t_{-i} \mid t_i) = \frac{p(t_{-i} \cap t_i)}{p(t_i)}$$

$$= \frac{p(t)}{\sum_{t'_{-i} \in T_{-i}} p(t'_1, \ldots, t'_{i-1}, t_i, t'_{i+1}, \ldots, t'_n)}$$

Ex Two players, two typs each (BofS)

Man

P1: y_1, n_1

ys no

Woman

P2: y_2, n_2

$p(t_1, t_2)$

		P2 y_2	P2 n_2
P1	y_1	2/6	2/6
P1	n_1	1/6	1/6

$$p(y_2 \mid y_1) = \frac{p(y_2 \cap y_1)}{p(y_1)} = \frac{p(y_1, y_2)}{p(y_1 \cap y_2) + p(y_1 \cap n_2)}$$

Bays's thm

$P(y_2 \mid y_1) = \frac{\quad}{P(y_1)} = \frac{\quad}{P(y_1 \cap y_2) + P(y_1 \cap n_2)}$ thm

$$= \frac{2/6}{2/6 + 2/6} = \frac{1}{2}$$

It's
Its

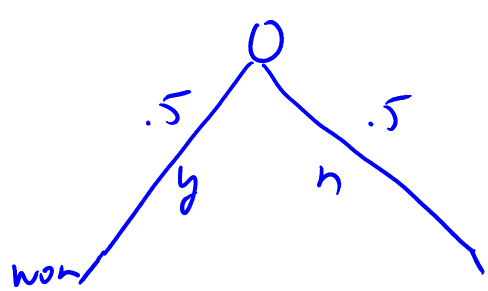
Given

		y_2	n_2
y_1	$\longrightarrow$	$1/2$	$1/2$
n_1	$\longrightarrow$	$1/2$	$1/2$

Given

		y_1	n_1
y_2	$\longrightarrow$	$2/3$	$1/3$
n_2	$\longrightarrow$	$2/3$	$1/3$

Ex. BoS one-sided incomplete info

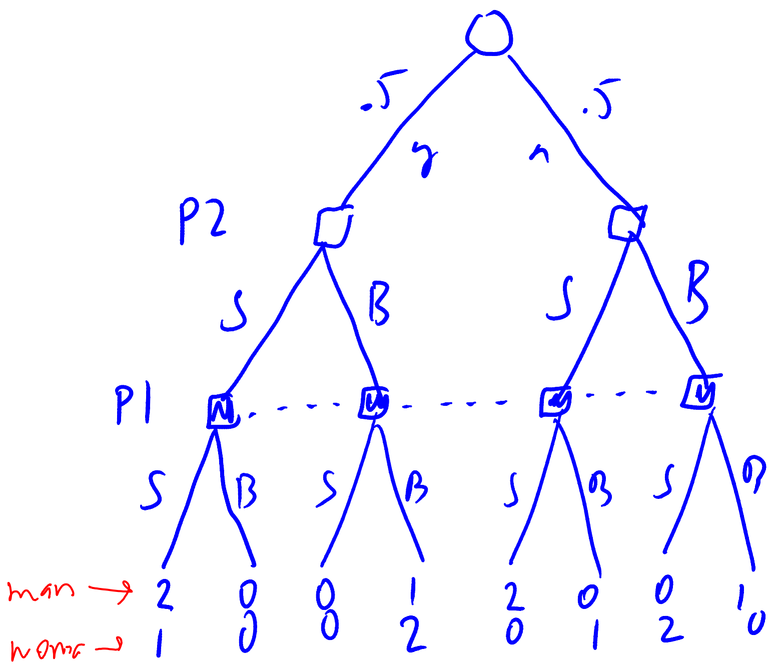


man

$$\begin{array}{c|cc} & S & B \\ \hline S & 2,1 & 0,0 \\ B & 0,0 & 1,2 \end{array} \qquad \begin{array}{c|cc} & S & B \\ \hline S & 2,0 & 0,2 \\ B & 0,1 & 1,0 \end{array}$$

woman(y)   woman(n)

original



$p(y|1)=\frac{1}{2}, \quad p(n|1)=\frac{1}{2}$

Type combinations: (1, y)
(1, n)

	y_1	y_2
1	•	•

Strategies

P1: S B

P2:

y	n
S	S
S	B
B	S

$$
\begin{array}{cc}
S & B \\
B & S \\
B & B
\end{array}
$$

$$
\begin{array}{ccccc}
 & \overset{y\,n}{SS} & \overset{y\,n}{SB} & \overset{y\,n}{BS} & \overset{y\,n}{BB} \\
P1 \quad S & 2,2 \quad 1,0 & 2,0 \quad 1,2 & 0,2 \quad 0,0 & 0,0 \quad 0,2 \\
B & 0,0 \quad 0,1 & 0,1 \quad 0,0 & 1,0 \quad 2,1 & 1,1 \quad 2,0
\end{array}
$$

$$
\begin{array}{ccccc}
 & \overset{y\,n}{SS} & \overset{y\,n}{SB} & \overset{y\,n}{BS} & \overset{y\,n}{BB} \\
S & \bar{2},(\bar{1},0) & \bar{1},(\bar{1},\bar{2}) & \bar{1},(0,0) & 0,(0,\bar{2}) \\
B & 0,(0,\bar{1}) & \tfrac{1}{2},(0,0) & \tfrac{1}{2},(\bar{2},\bar{1}) & \bar{1},(\bar{2},0)
\end{array}
$$

BNE: $(S, \overset{y\,n}{SB})$

Soccer

Soccer if y

ballet if n