The normal probability distribution

- A normal random variable X has the distribution (density)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty < x < \infty$$

- Here μ is the mean and σ is the standard deviation, and $\pi = 3.14159\ldots$ and $e = 2.71828\ldots$

- We write $X \sim N(\mu, \sigma^2)$

- Calculating probabilities corresponds to finding the area under the curve defined by $f(x)$

- If we use tables, it is necessary to standardize X as follows

$$Z = \frac{X - \mu}{\sigma}$$

- We have that $Z \sim N(0, 1)$.

- This way we can find any probability $\Pr(a < X < b)$ by converting the X problem into Z problem, i.e.,

$$\begin{aligned} \Pr(a < X < b) &= \Pr\left(\frac{a-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{b-\mu}{\sigma}\right) \\ &= \Pr\left(\frac{a-\mu}{\sigma} < Z < \frac{b-\mu}{\sigma}\right) \end{aligned}$$

- Tables have the probabilities for Z r.v.

- Of course, R will do it much more easily.

- In our example, $X \sim N(7.13, 0.27^2)$ and we need to find

$$\begin{aligned} \Pr(6.75 < X < 7.49) &= \Pr(-1.4074 < Z < 1.3333) \\ &= 0.8291 \end{aligned}$$