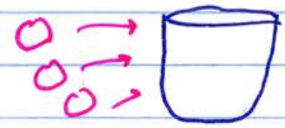
c) Binomial distribution

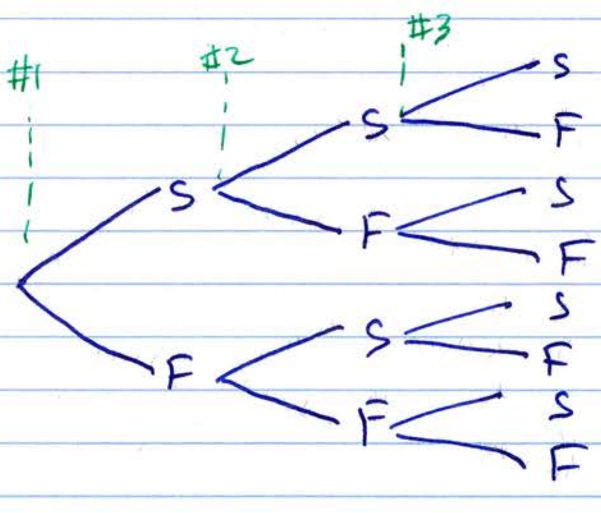
Suppose I have 60% success rate in my throws of tennis balls in a bucket

If I throw 3 balls, what's the prob. that I will get in 0, 1, 2 or 3 in the bucket?



Some observations
- 3 identical trials (throws)
- Two possible outcomes
  - success (in bucket)
  - failure (out of ")
- Pr(success) = 0.6 = p
- Throws are independent

Pr(failure) = 1-p, = 0.4



1 2 3	# of S
SSS	3
SSF	2
SFS	2
SFF	1
FSS	2
FSF	1
FFS	1
FFF	0

Independence: $Pr(\text{all } S) = Pr(SSS) = .6 \times .6 \times .6 = (.6)^3 = .216 = p^3$

$$Pr(2\ S) = Pr(SSF) + Pr(SFS) + Pr(FSS)$$
$$= p^2(1-p) + p^2(1-p) + p^2(1-p)$$
$$= 3(.6)^2(.4) = 3(.144) = .432$$

Note: The 8 outcomes are mutually exclusive & collectively exhaustive

How many different ways can I get 0, 1, 2 or 3 successes?
(Look at pink outcomes)

0 success	1 way (FFF)
1 "	3 "
2 "	3 "
3 "	1 " (SSS)

Is there an easy way to calculate these numbers?
YES!

Suppose we have n trials and want to find the # ways we can get x successes

Formula is $\binom{n}{x} = \frac{n!}{x!(n-x)!}$ where $n! = n(n-1)\cdots 2\cdot 1$

"n choose x"

$0! = 1$
$1! = 1$
$2! = 2\cdot 1 = 2$
$3! = 3\cdot 2\cdot 1 = 6$
$\vdots$

Our case: $n=3$

$$\left.\begin{aligned}\binom{3}{3} &= \frac{3!}{3!\,0!} = 1\\ \binom{3}{2} &= \frac{3!}{2!\,1!} = 3\\ \binom{3}{1} &= \frac{3!}{1!\,2!} = 3\\ \binom{3}{0} &= \frac{3!}{0!\,3!} = 1\end{aligned}\right\}\text{as we saw above}$$

Pascal's triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
etc.

Comment: Lotto 6/49: $\binom{49}{6} = 13,983,816$

c

How do we find the probabilities now?

$$\boxed{p(x) = Pr(x \text{ success in } n \text{ trials}) = \binom{n}{x} p^x (1-p)^{n-x} \qquad x = 0, 1, 2, \ldots, n}$$

Binomial distribution

Our case $n = 3$, $x = 0, 1, 2, 3$

$$p(x) = Pr(x \text{ in } 3) = \binom{3}{x} (0.6)^x (0.4)^{3-x}, \quad x = 0, 1, 2, 3$$

$p(0) = \binom{3}{0} (0.6)^0 (0.4)^3 = 1 \cdot 1 \cdot (0.064) = 0.064$

$p(1) = \binom{3}{1} (0.6)^1 (0.4)^2 = \qquad = 0.282$

$p(2) = \qquad = 0.432$

$p(3) = \qquad = 0.216$

$\qquad 1.000$

In general, if X is binomial with (n, p), then

$$E(X) = \mu_X = np$$

$$\sigma_X^2 = np(1-p)$$

$$\sigma_X = \sqrt{np(1-p)}$$

Ex. Our case: $n = 3$, $p = 0.6$, $E(X) = 1.8$

$\sigma_X^2 = 0.72$

$\sigma_X = 0.84$

- MegaStat ex.