

## Hierarchical methods

- As mentioned above, agglomerative hierarchic methods start with a matrix of ‘distances’ between individuals. All individuals begin alone in groups of one and groups that are ‘close’ together are merged. (Rcmdr recognizes several different measures of distance.)
- There are various ways to define ‘close’. The simplest is in terms of nearest neighbours (single linkage). For example, suppose there is the following distance matrix for five objects as shown in Table 1:

|          | <i>1</i> | <i>2</i> | <i>3</i> | <i>4</i> | <i>5</i> |
|----------|----------|----------|----------|----------|----------|
| <i>1</i> | 0        |          |          |          |          |
| <i>2</i> | 2        | 0        |          |          |          |
| <i>3</i> | 6        | 5        | 0        |          |          |
| <i>4</i> | 10       | 9        | 4        | 0        |          |
| <i>5</i> | 9        | 8        | 5        | 3        | 0        |

Table 1: Distance matrix.

- The calculations are then as shown in Table 2. Groups are merged at a given level of distance if one of the individuals in one group is that distance or closer to at least one individual in the second group.
- At a distance of 0 all five objects are on their own. The distance matrix shows that the smallest distance between two objects is 2, between the first and second objects. Hence at a distance level of 2 there are four groups (1,2), (3), (4) and (5).
- The next smallest distance between objects is 3, between objects 4 and 5. Hence at a distance of 3 there are three groups (1,2), (3) and (4,5).

| <b>Distance</b> | <b>Clusters</b> |
|-----------------|-----------------|
| 0               | 1, 2, 3, 4, 5   |
| 2               | (1,2), 3, 4, 5  |
| 3               | (1,2), 3, (4,5) |
| 4               | (1,2), (3,4,5)  |
| 5               | (1, 2, 3, 4, 5) |

Table 2: Groupings (clusters).

- The next smallest distance is 4, between objects 3 and 4. Hence at this level of distance there are two groups (1,2) and (3,4,5).
- Finally, the next smallest distance is 5, between objects 2 and 3 and between objects 3 and 5. At this level the two groups merge into the single group (1, 2, 3, 4, 5) and the analysis is complete.

## **Calculations and the dendrograms**

The calculations leading to Table 2 and the resulting dendrogram are shown in Figures 1 and 2 which illustrates how agglomeration takes place.

Calculation of steps from 5 clusters to 1 cluster

|   | 1  | 2 | 3 | 4 | 5 |                                  |
|---|----|---|---|---|---|----------------------------------|
| 1 | 0  |   |   |   |   |                                  |
| 2 | ②  | 0 |   |   |   | $\min_{i,k} d_{ik} = d_{12} = 2$ |
| 3 | 6  | 5 | 0 |   |   |                                  |
| 4 | 10 | 9 | 4 | 0 |   |                                  |
| 5 | 9  | 8 | 5 | 3 | 0 | New cluster = (1,2)              |

|      | (12)                                     | 3 | 4 | 5 |  |
|------|--|---|---|---|--|
| (12) | 0  |   |   |   |  |
| 3    | $\min(d_{31}, d_{32}) = \min(6, 5) = 5$  | 0 |   |   |  |
| 4    | $\min(d_{41}, d_{42}) = \min(10, 9) = 9$ | 4 | 0 |   |  |
| 5    | $\min(d_{51}, d_{52}) = \min(9, 8) = 8$  | 5 | ③ | 0 |  |

New cluster = (4,5)

|      | (12)   | (45)                                    | 3 |   |
|------|--|---|---|---|
| (12) | 0  |   |   |   |
| (45) | $\min(d_{41}, d_{42}, d_{51}, d_{52}) = \min(10, 9, 9, 8) = 8$ | 0                                       |   | 0 |
| 3    | $\min(d_{31}, d_{32}) = \min(6, 5) = 5$                        | $\min(d_{34}, d_{35}) = \min(4, 5) = 4$ |   | 0 |

New cluster (345)

|       | (12)                          | (345) | Final cluster |
|-------|-------------------------------|-------|---------------|
| (12)  | 0                             |       | (12345)       |
| (345) | $\min(6, 5, 10, 9, 9, 8) = 5$ | 0     |               |

Figure 1: Calculations.

The sequence of steps leading from 5 clusters to 1 cluster

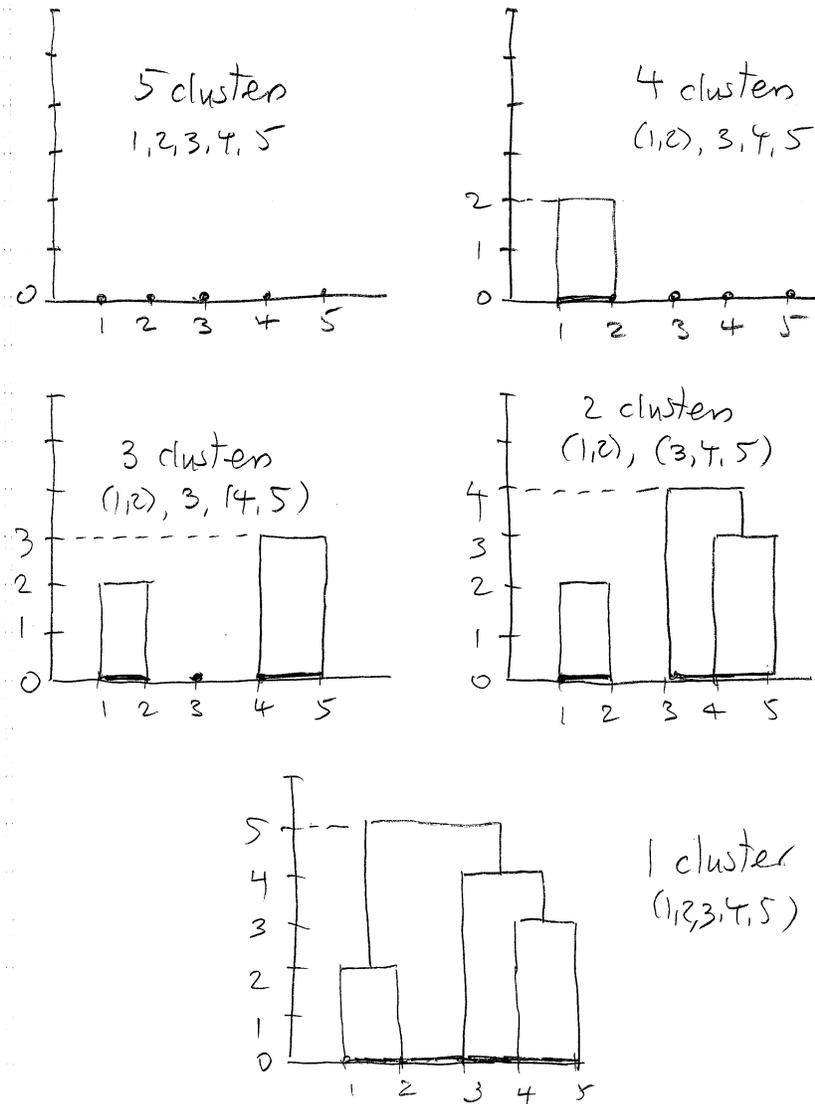


Figure 2: Dendrogram.